438(4): New m Theory of Light Deflection due to Gravitation

Consider a photon of mass m approaching an object of mass M. In m theory the equations of motion of the photon are:

$$\frac{dH}{dt} = 0 \; , \quad \frac{dL}{dt} = 0 \qquad -(1)$$

where

$$H = m(r)\, \gamma m c^2 - m(r)^{1/2} \frac{m M G}{r} \qquad -(2)$$

and

$$L = \frac{\gamma m r^2}{m(r)} \dot{\phi} . \qquad -(3)$$

Here:

$$\gamma = \left(m(r) - \frac{\dot{r}^2 + r^2 \dot{\phi}^2}{m(r)\, c^2} \right)^{-1/2} \qquad -(4)$$

The photon mass m is very small but identically non-zero. So to compute the orbit of the photon of mass m, Eqs. (1) to (4) must be integrated numerically under the condition:

$$m \ll M \; ; \; m \neq 0 \qquad -(5)$$

and

$$m \rightarrow 0 . \qquad -(6)$$

In the Newtonian theory of light deflection, Eqs. (1) to (4) reduce to:

$$m\ddot{r} - m r \dot{\phi}^2 = -\frac{m M G}{r^2} \qquad -(7)$$

and

$$m\left(r\ddot{\phi} + 2\dot{\phi}\dot{r} \right) = 0, \qquad -(8)$$

The orbital velocity is:

$$v_N^2 = MG \left(\frac{2}{r} - \frac{1}{a} \right) \qquad -(9)$$

and

$$r = \frac{\alpha}{1 + \epsilon \cos\phi} \qquad -(10)$$

The angle of deflection is:

$$\Delta\phi = \sin^{-1} \frac{1}{\epsilon} \qquad -(11)$$

Eq. (7) can be written as:

2)
$$\frac{d^2}{d\phi^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{1}{\alpha} \quad -(12)$$

which is the Binet equation.

For the ellipse: $0 < \epsilon < 1 \quad -(13)$

and for the circle: $\epsilon = 0 \quad -(14)$

The half right latitude is defined as:

$$\alpha = \frac{L^2}{m^2 M G} \quad -(15)$$

and the eccentricity as:

$$\epsilon^2 = 1 + \frac{2HL^2}{m^3 M^2 G^2} \quad -(16)$$

The hamiltonian is:

$$H = \frac{1}{2}mv^2 - \frac{mMG}{r} \quad -(17)$$

and using eq. (9)

$$H = -\frac{1}{2}\frac{mMG}{a} \quad -(18)$$

From eqs. (16) and (18):

$$\epsilon^2 = 1 - \frac{1}{2}\frac{mMG}{a}\frac{HL^2}{m^3 M^2 G^2}$$

$$= 1 - \frac{1}{a}\left(\frac{L^2}{m^2 MG}\right)$$

$$= 1 - \frac{\alpha}{a} \quad -(19)$$

So for the ellipse:

$$a = \frac{\alpha}{1-\epsilon^2}, \quad -(20)$$

$-(21)$

3) For the circle:

$$\epsilon = 0, \quad -(22)$$

So

$$\alpha = a \cdot \quad -(23)$$

For the hyperbola:

$$\epsilon^2 := 1 + \frac{2|H|L^2}{m^3 M^2 G^2} \quad -(24)$$

$$= 1 + \frac{\alpha}{a}$$

and

$$\epsilon > 1 \quad -(25)$$

So

$$a = \frac{\alpha}{\epsilon^2 - 1} \quad -(26)$$

Summary of the Newtonian Trajectory of the Photon

This is the conic section:

$$r = \frac{\alpha}{1 + \epsilon \cos\phi} \quad -(27)$$

and for the ellipse:

$$0 < \epsilon < 1, \quad \frac{\alpha}{a} = 1 - \epsilon^2, \quad -(28)$$

for the hyperbola:

$$\epsilon > 1, \quad \frac{\alpha}{a} = 1 + \epsilon^2 \quad -(29)$$

for the circle:

$$\epsilon = 0, \quad \frac{\alpha}{a} = 1 \quad -(30)$$

In these equations a is the semi major axis.

The conventional theory of light deflection due to gravitation is based on:

$$v_N^2 = MG\left(\frac{2}{R_0} - \frac{1}{a}\right) \quad -(31)$$

+) and assumes a hyperbolic trajectory of large eccentricity,

So $$a = \frac{\alpha}{1-\epsilon^2}, \quad R_0 = \frac{\alpha}{1+\epsilon} \quad -(32)$$

It follows that:
$$v_N^2 = \frac{mG}{R_0}(\epsilon + 1). \quad -(33)$$

Note carefully that for the hyperbola, a is defined as < 0.

Finally, if the eccentricity of the hyperbola is:
$$\epsilon \gg 1 \quad -(34)$$

as in light grazing a star, then
$$\epsilon \sim \frac{R_0 v_N^2}{mG} \quad -(35)$$

In the usual Newtonian theory of light deflection:
$$\Delta\phi = 2\sin^{-1}\frac{1}{\epsilon} = 2\sin^{-1}\frac{mG}{R_0 v_N^2} \quad -(36)$$

i.e.
$$\Delta\phi \sim \frac{2mG}{R_0 v_N^2} \quad -(37)$$

In UFT 419 this theory was extended to m theory.

The experimental result is:
$$\Delta\phi = \frac{4mG}{R_0 c^2} \quad -(38)$$

In earlier UFT papers the result (38) was obtained from the Newtonian (37) using the relativistic velocity of special relativity:
$$v^2 = \frac{v_N^2}{1 - \frac{v_N^2}{c^2}} \quad -(39)$$

so
$$v_N^2 = \frac{v^2}{\ldots} \rightarrow \frac{c^2}{\ldots} \quad -(40)$$

5) as $v \to c$ — (41)

The experimental result (38) is obtained exactly from eq. (40). This means that there is an upper bound of $c^2/2$ on the Newtonian velocity, regarded as a parameter in

$$v = \gamma v_N \quad -(42)$$

This theory can now be improved by solving eqs. (1) to (6) without using the Newtonian approximation at all. This would be an extension of the calculations carried out in 438(3) for the dark star