Orbit around a Heavy Mass is to Thay

will refer to UFT149 and earlier papers to Thay

dynamics is Thay are governed by the two fundamental
conservation equations:

\[ \frac{dH}{dt} = 0 \quad - (1) \]
\[ \frac{dL}{dt} = 0 \quad - (2) \]

where \( H \) and \( L \) are the Hamiltonian and angular

while \( H \) and \( L \) appear

\[ H = m(r) Vmc^2 - m(r) \frac{MmG}{r} \quad - (3) \]
\[ L = \frac{m(r) Vmc^2}{\phi} \quad - (4) \]

Here \( V \) is the generalized Lorentz factor:

\[ V = \left( m(r) - \frac{r^2 + r^2 \phi}{m(r)c^2} \right)^{-1/2} \quad - (5) \]

and the potential energy is:

\[ U = -m(r) \frac{MmG}{r} \quad - (6) \]

In the plane polar coordinate system \((r, \phi)\), in

the plane polar coordinate system \((r, \phi)\), the mass \( m \) as the mass \( M \) in a

plane, separated by a distance \( r \), and \( G \) is Newton's

constant.

Note carefully that these equations do not

depend on the Einstein field equation.

Using computer algebra it can be shown that:
\[
\ddot{r} - \frac{\ddot{r}}{r} = \frac{\text{d}m(r)}{\text{d}r} \left( \frac{c^2 m(r)}{r} + \frac{m_6}{2 Y^3 \gamma^2 \gamma m(r)^{1/2}} - \frac{3c^2}{2Y^2} \right)
\]

\[
- \frac{1}{m(r)} \frac{\text{d}m(r)}{\text{d}r} \left( \frac{2 - M_6}{2 Y c \gamma m(r)^{1/2}} \right) + \frac{M_6}{Y c \gamma m(r)^{1/2}} - \frac{\ddot{r}}{r}
\]

The mass of \( m \) about \( M \) is obtained by solving eqs. (7) and (8) numerically.

In the Newtonian limit they reduce to:

\[
\ddot{r} - \frac{\ddot{r}}{r} = - \frac{M_6}{r^2}
\]

and

\[
\ddot{r} + 2 \ddot{\phi} r = 0
\]

To simulate the investigation it is possible to increase \( M \) to near infinity, and first solve eqs. (9) and (10) numerically for instance. The solutions to eqs. (9) and (10) are given by

\[
r = \frac{d}{\cos \phi}
\]

where \( d \) is the half light ray at \( \phi = \frac{\pi}{2} \), and \( e \) is the eccentricity.

The Hamiltonian for eqs. (9) and (10) is:

\[
H = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\phi}^2) - \frac{m M_6}{r}
\]
and the specific angular momentum is:
\[ L = m s^2 \phi \] - (13)

The half light latitude is:
\[ \lambda = \frac{L}{m^2 M s^2} \] - (14)

and the eccentricity is given by:
\[ e^2 = 1 + \frac{2 H L}{m^2 M s^2} \] - (15)

The orbital velocity is:
\[ v^2 = M s^2 \left( \frac{a}{r} - \frac{1}{a} \right) \] - (16)

where
\[ a = \frac{\lambda}{1 - e^2} \] - (17)

For an elliptical orbit:
\[ 0 < e < 1 \] - (18)

and for hyperbola:
\[ e > 1 \] - (19)

Using these equations it is possible to graph the orbit as:
\[ M \to \infty \] - (20)

If \( m \) remain finite,
From eq. (14), \[ \lambda \to 0 \] as \( M \to \infty \) - (21)

From eq. (15), \[ e \to 1 \] as \( M \to \infty \) - (22)

From eq. (11), \[ r \to 0 \] as \( M \to \infty \) - (23)
(1) \( \text{For eq. } (16): \quad v \rightarrow \infty \) \\
\( M \rightarrow \infty \)

So the ellipsoidal body shrinks to a point and the orbital velocity of \( m \) about \( M \) approaches infinity. These characteristics could be graphed and/or animated, and are independent of mass \( m \), becoming at rather short distances, not apparent in the equations of motion (9) and (10).

The equations are true for all photons of mass \( m \). This is a beam of light, captured by an observer in space in the vicinity of \( M \), so an observation of space in the vicinity of \( M \) would produce a dark area.

All the characteristics of a "black hole" can be reproduced by graphics based on the above Newtonian equations. Once the light is trapped by the pseudo-gravitational mass \( M \), its escape velocity is given by:

\[
\frac{1}{2} mv^2 = \frac{MM_0}{r} - (25)
\]

\[
v = \left( \frac{2MM_0}{r} \right)^{1/2} - (26)
\]

So in Newtonian dynamics the light is trapped and can never escape.

When the complete equations (7) and (8) are considered, it is to the next note, a variety of orbital behaviors become possible, notably precession, as is in UFT 419. This will be considered in the next note.