30(5): The m Theory of Vacuum Polarization.

Vacuum polarization can be thought of as the screening of a point charge by a dielectric medium, in this case the vacuum. The usual theory of vacuum polarization asserts that virtual pair production generates a vacuum polarization and magnetization, but this has never been observed directly.

Using google keywords "Measurement of Vacuum Polarization", third and fifth sites, shows that vacuum polarization can be thought of as modifying the Coulombic potential to:

$$\phi(r) = -\frac{e}{4\pi\epsilon_0 r}\left(1 - \frac{2\alpha}{3\pi}\log_e\left(\frac{r}{\lambda_c}\right) + \cdots\right) \quad -(1)$$

for:

$$\frac{r}{\lambda_c} \ll 1 \quad -(2)$$

and

$$\phi(r) = -\frac{e}{4\pi\epsilon_0 r}\left(1 + \frac{\alpha}{4\sqrt{\pi}}\left(\frac{r}{\lambda_c}\right)^{-3/2} e^{-2r/\lambda_c} + \cdots\right) \quad -(3)$$

for

$$\frac{r}{\lambda_c} \gg 1. \quad -(4)$$

Here:

$$\lambda_c = \frac{\hbar}{mc} = 3.86 \times 10^{-13}\ \text{m} \quad -(5)$$

and α is the fine structure constant:

$$\alpha = \frac{e^2}{4\pi\hbar c\epsilon_0} = 0.007297351 \quad -(6)$$

In m theory:

$$\phi(r) = -\, m(r)^{1/2} \frac{e}{4\pi\epsilon_0 r} \quad -(7)$$

so vacuum polarization is explained immediately by the premultiplier $m(r)^{1/2}$. If the results of Q.E.D. are accepted for the sake of argument, then $m(r)^{1/2}$ for vacuum polarization is given by:

$$m(r)^{1/2} = 1 - \frac{2\alpha}{3\pi} \log_e\left(\frac{r}{\lambda_c}\right) + \ldots \quad -(8)$$

for

$$\frac{r}{\lambda_c} \ll 1 \quad -(9)$$

and

$$m(r)^{1/2} = 1 + \frac{\alpha}{4\sqrt{\pi}}\left(\frac{r}{\lambda_c}\right)^{-3/2} e^{-2r/\lambda_c} + \ldots \quad -(10)$$

for

$$\frac{r}{\lambda_c} \gg 1. \quad -(11)$$

It is seen that the Coulomb law is changed by vacuum polarization.

In classical electrodynamics the vacuum charge density $\rho(\text{vac})$ is introduced, so the Coulomb law becomes:

$$\underline{\nabla} \cdot \underline{E} = \rho_{vac} / \epsilon_0 \quad -(12)$$

and the Ampère Maxwell law becomes:

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{J}(\text{vac}) + \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} \quad -(13)$$

3) where $\underline{J}(vac)$ is the vacuum current. Eq. (13) can be expressed as:

$$\underline{\nabla} \times \underline{H} = \underline{J}(vac) + \frac{\partial \underline{D}}{\partial t} \quad -(14)$$

where:

$$\underline{D} = \epsilon_0 \underline{E} + \underline{P}(vac) \quad -(15)$$

and

$$\underline{B} = \mu_0 \left(\underline{H} + \underline{M}(vac) \right) \quad -(16)$$

Here $\underline{P}(vac)$ is the vacuum polarization and $\underline{M}(vac)$ the vacuum magnetization. If there is no vacuum polarization or vacuum magnetization then:

$$\underline{D} = \epsilon_0 \underline{E} \; ; \; \underline{B} = \mu_0 \underline{H} \quad -(17)$$

So eq. (14) becomes:

$$\frac{1}{\mu_0} \underline{\nabla} \times \underline{B} = \underline{J}(vac) + \epsilon_0 \frac{\partial \underline{E}}{\partial t} \quad -(18)$$

which is eq. (13) using:

$$\mu_0 \epsilon_0 = \frac{1}{c^2} \quad -(19)$$

Note carefully that the concept of $\rho(vac)$ and $\underline{J}(vac)$ are contained in the ECE2 equations of electrodynamics, which are based directly on the Cartan identity and its Hodge generally covariant dual. As in UFT317,

$$\rho(vac) = \epsilon_0 \underline{\kappa} \cdot \underline{E}(vac) \quad -(20)$$

$$\underline{J}(vac) = \frac{1}{\mu_0} \underline{\kappa} \times \underline{B}(vac) \quad -(21)$$

where $\underline{E}(vac)$ and $\underline{B}(vac)$ are the vacuum electric

) field strength and vacuum magnetic flux density, and where the tetrad four vector is:

$$q^{\mu} = (q^{0}, \underline{q}) \quad -(22)$$

and spin connection four vector is:

$$\omega^{\mu} = (\omega^{0}, \underline{\omega}) \quad -(23)$$

The four vector κ^{μ} is defined as:

$$\kappa^{\mu} = \frac{2 q^{\mu}}{r^{(0)}} - \kappa^{\mu} \quad -(24)$$

where $r^{(0)}$ is a characteristic length.

So in ECE2 theory the vacuum charge and current originate in the vacuum fields $\underline{E}$(vac) and $\underline{B}$(vac). In this theory the Q.E.D. correction to the Coulomb law is reproduced exactly by eqs. (8) and (10).

In Q.E.D. the g factor of the electron is given by:

$$g = 2 + \frac{\alpha}{\pi} + O(\alpha^{2}) + \ldots \quad -(25)$$

From Note 429(1), the g factor of the electron is

$$g = \frac{2}{n(r)^{1/2}} \quad -(26)$$

so

$$\frac{2}{n(r)^{1/2}} = 2 + \frac{\alpha}{\pi} + \ldots \quad -(27)$$

and

$$n(r)^{1/2} = \frac{2}{2 + \frac{\alpha}{\pi} + \ldots} \quad -(28)$$

so $n(r) = 0.99942$ $-(29)$

) The $m(r)$ function can be calculated from computer algebra using eqs. (8) to (11) and plotted against r. At a given value of r, $m(r)$ from eqs. (8) and (10) will be the same as $m(r)$ from eq. (28). This point r will be related to the electron radius. It could be argued that the electron radius is a minimum for eq. (8) and a maximum for eq. (9).

In the second order theory of the g factor:

$$g = \frac{4}{\left(\frac{\hbar\omega}{mc^2} + m(r)\right)^{1/2}} = 2 + \frac{\alpha}{\pi} + \dots \quad (30)$$

and $m(r)$ can be calculated in terms of the angular frequency ω of the electron. For the rest electron:

$$\hbar\omega_0 = mc^2 \quad (31)$$

so

$$\frac{4}{\left(1 + m(r)\right)^{1/2}} = 2 + \frac{\alpha}{\pi} = 2.002323 \quad (32)$$

and

$$m(r)^{1/2} = 0.99768 \quad (33)$$

$$m(r) = 0.99884 \quad (34)$$

for the rest electron. For a moving electron this value will change.

Finally the Lamb shift in quantum electrodynamics is a change in potential energy:

$$\langle \Delta V \rangle = \alpha^5 mc^2 \frac{1}{6\pi} \log_e\left(\frac{1}{\pi\alpha}\right) \quad (35)$$

between the $2P_{1/2}$ and $2S_{1/2}$ states of atomic H.

6) Here ΔV is defined by:

$$\Delta V = \bar{V}(\underline{r} + \delta\underline{r}) - V(\underline{r}) \quad - (36)$$

$$= \frac{1}{6} \left\langle (\delta r)^2 \right\rangle_{vac} \left\langle \nabla^2 \left(-\frac{e^2}{4\pi\epsilon_0 r} \right) \right\rangle_{atom}$$

For the Coulomb potential:

$$\left\langle \nabla^2 \left(\frac{-e^2}{4\pi\epsilon_0 r} \right) \right\rangle = -\frac{e^2}{4\pi\epsilon_0} \int \psi^* \nabla^2 \left(\frac{1}{r} \right) \psi \, dr$$

$$= \frac{e^2}{\epsilon_0} \left| \psi(0) \right|^2 \quad - (37)$$

For s orbitals:

$$\psi_{2S}(0) = \frac{1}{(8\pi a_0^3)^{1/2}} \quad - (38)$$

so

$$\left\langle \nabla^2 \left(-\frac{e^2}{4\pi\epsilon_0 r} \right) \right\rangle = \frac{e^2}{8\pi\epsilon_0 a_0^3} \quad - (39)$$

For p orbitals this expectation value vanishes. $\quad - (40)$

In QED:

$$\left\langle (\delta r)^2 \right\rangle_{vac} \sim \frac{1}{2\epsilon_0 \pi^2} \left(\frac{e^2}{\hbar c} \right) \left(\frac{\hbar}{mc} \right)^2 \log_e \frac{4\epsilon_0 \hbar c}{e^2}$$

so $\langle \Delta V \rangle$ is given by eq. (35) in Q.E.D.

In my theory the Lamb shift is calculated from:

$$E_n = -\frac{\hbar^2}{2m} \left(\int \psi^* \frac{1}{m(r)^{1/2}} \nabla^2 \psi \, d\tau + \int \psi^* \underline{\nabla} \left(\frac{1}{m(r)^{1/2}} \right) \cdot \underline{\nabla} \psi \, d\tau \right)$$

$$- \frac{e^2}{4\pi\epsilon_0} \int \psi^* \frac{m(r)^{1/2}}{r} \psi \, d\tau \quad - (41)$$

and $\underline{m(r)}$ adjusted to give the result (35).

) Note that the third term in eq. (41) is the
expectation value of :

$$V = -\frac{e^2}{4\pi\epsilon_0 r} n(r)^{1/2} \quad -(42)$$

which is the vacuum polarization term. So the
Lamb shift contains a contribution from vacuum polarization
as is well known.

In summary, the complicated and fairly
criticized method of quantum electrodynamics can be
replaced by a ~~theory~~.