```
(%i1)  kill(all);
(%o0)  done
```

```
(%i1)  assume(c>0, m>0, q_r>0);
(%o1)  [c>0,m>0,q_r>0]
```

Relativistic theory

1 Relativistic Hamilton equations II
inertial frame
H defined by velocity p_r^2/m^2

```
(%i2)  H: (p_r^2*c^2+m^2*c^4)^(1/2)-m*M*G/q_r;
```

$$(H)\quad \sqrt{c^2\,p_r^{\,2}+c^4\,m^2}-\frac{G\,M\,m}{q_r}$$

1.1 First Hamilton equations

```
(%i3)  H1: q_rd = diff(H,p_r);
```

$$(H1)\quad q_{rd}=\frac{c^2\,p_r}{\sqrt{c^2\,p_r^{\,2}+c^4\,m^2}}$$

```
(%i4)  H2: q_phid = diff(H,p_phi);
```

$$(H2)\quad q_{phid}=0$$

1.2 Second Hamilton equations

```
(%i5)  H3: p_rd = (-diff(H,q_r));
```

$$(H3)\quad p_{rd}=-\frac{G\,M\,m}{q_r^{\,2}}$$

```
(%i6)  H4: p_phid = -diff(H,q_phi);
```

$$(H4)\quad p_{phid}=0$$

not applicable

1.3 Rewrite equations

```
(%i7)  gamma1: (1-p_r^2/(m^2*c^2))^(-1/2);
```

$$(gamma1)\quad \frac{1}{\sqrt{1-\frac{p_r^{\,2}}{c^2\,m^2}}}$$

```
(%i8)  gamma: m*c*(m^2*c^2-p_r^2)^(-1/2);
```

$$(gamma)\quad \frac{c\,m}{\sqrt{c^2\,m^2-p_r^{\,2}}}$$

```
(%i9)  ratsimp(gamma-gamma1);
(%o9)  0
```

```
(%i10)  H1a: ratsubst(%gamma, gamma, H1);
```

$$(H1a)\quad q_{rd}=\frac{c^2\,p_r}{\sqrt{c^2\,p_r^{\,2}+\gamma^2\,c^2\,(c^2\,m^2-p_r^{\,2})}}$$

```
(%i11)  H1b: q_rd=(c^2*p_r)/sqrt(c^2*p_r^2*(1-%gamma^2)+%gamma^2*c^4*m^2);
```

$$(H1b)\quad q_{rd}=\frac{c^2\,p_r}{\sqrt{(1-\gamma^2)\,c^2\,p_r^{\,2}+\gamma^2\,c^4\,m^2}}$$

```
(%i12)  ratsimp(H1a-H1b);
(%o12)  0=0
```

(%i13) `H1c: q_rd=(c^2*p_r)/(c*sqrt(1^2*p_r^2*(1-%gamma^2)+%gamma^2*c^2*m^2));`

(H1c)
$$q_{rd}=\frac{c\,p_r}{\sqrt{(1-\gamma^2)\,p_r^{\,2}+\gamma^2\,c^2\,m^2}}$$

(%i14) `ratsimp(radcan(H1b)-radcan(H1c));`

(%o14) $0=0$

1.4 Resolve for p_r

(%i15) `H1d: H1^2/p_r^2;`

(H1d)
$$\frac{q_{rd}^{\,2}}{p_r^{\,2}}=\frac{c^4}{c^2\,p_r^{\,2}+c^4\,m^2}$$

(%i16) `H1e: 1/H1d;`

(H1e)
$$\frac{p_r^{\,2}}{q_{rd}^{\,2}}=\frac{c^2\,p_r^{\,2}+c^4\,m^2}{c^4}$$

(%i17) `H1d: solve(H1e, p_r^2);`

(H1d)
$$[\,p_r^{\,2}=-\frac{c^2\,m^2\,q_{rd}^{\,2}}{q_{rd}^{\,2}-c^2}\,]$$

Rewrite with sign exchange in denominator

(%i18) `H1e: p_r^2 = -num(rhs(first(H1d)))/-denom(rhs(first(H1d)));`

(H1e)
$$p_r^{\,2}=\frac{c^2\,m^2\,q_{rd}^{\,2}}{c^2-q_{rd}^{\,2}}$$

(%i19) `H1f: sqrt(H1e);`

(H1f)
$$\left|p_r\right|=\frac{c\,m\left|q_{rd}\right|}{\sqrt{c^2-q_{rd}^{\,2}}}$$

This is p_r = gamma m q_r dot

2 Relativistic Hamilton equations IV general frame H defined by velocities (p_r^2 + p_phi^2/q_r^2)/m^2

(%i20) `H: ((p_r^2+p_phi^2/q_r^2)*c^2+m^2*c^4)^(1/2)-m*M*G/q_r;`

(H)
$$\sqrt{c^2\left(\frac{p_{phi}^{\,2}}{q_r^{\,2}}+p_r^{\,2}\right)+c^4\,m^2}-\frac{G\,M\,m}{q_r}$$

2.1 First Hamilton equations

(%i21) `H1: q_rd = diff(H,p_r);`

(H1)
$$q_{rd}=\frac{c^2\,p_r}{\sqrt{c^2\left(\frac{p_{phi}^{\,2}}{q_r^{\,2}}+p_r^{\,2}\right)+c^4\,m^2}}$$

(%i22) `H2: q_phid = diff(H,p_phi);`

(H2)
$$q_{phid}=\frac{c^2\,p_{phi}}{\sqrt{c^2\left(\frac{p_{phi}^{\,2}}{q_r^{\,2}}+p_r^{\,2}\right)+c^4\,m^2\,q_r^{\,2}}}$$

2.2 Second Hamilton equations

(%i23)
```
H3: p_rd = (-diff(H,q_r));
```

(H3)
$$p_{rd}=\frac{c^2\,p_{phi}^{\,2}}{\sqrt{c^2\left(\frac{p_{phi}^{\,2}}{q_r^{\,2}}+p_r^{\,2}\right)+c^4\,m^2\,q_r^{\,3}}}-\frac{G\,M\,m}{q_r^{\,2}}$$

(%i24)
```
H4: p_phid = -diff(H,q_phi);
```

(H4)
$$p_{phid}=0$$

2.3 Rewriting with "E factor"

(%i25)
```
E1: 1/sqrt(c^2*(p_phi^2/q_r^2+p_r^2)+c^4*m^2);
```

(E1)
$$\frac{1}{\sqrt{c^2\left(\frac{p_{phi}^{\,2}}{q_r^{\,2}}+p_r^{\,2}\right)+c^4\,m^2}}$$

(%i29)
```
ratsubst(E_1, E1, H1);
ratsubst(E_1, E1, H2);
expand(ratsubst(E_1, E1, H3));
ratsubst(E_1, E1, H4);
```

(%o26)
$$q_{rd}=E_1\,c^2\,p_r$$

(%o27)
$$q_{phid}=\frac{E_1\,c^2\,p_{phi}}{q_r^{\,2}}$$

(%o28)
$$p_{rd}=\frac{E_1\,c^2\,p_{phi}^{\,2}}{q_r^{\,3}}-\frac{G\,M\,m}{q_r^{\,2}}$$

(%o29)
$$p_{phid}=0$$