437(3): Quantization Schemes for m Theory

On a fully relativistic level the quantization could be based on:

$$E^2 = m(r_1)\left(p_1^2 c^2 + m^2 c^4\right) \quad -(1)$$

using a Dirac type procedure. In an approximate scheme the hamiltonian can be defined by:

$$H = m(r_1)\,\gamma m c^2 - \frac{mMG}{r_1} \quad -(2)$$

where

$$\gamma = \left(m(r_1) - \frac{v_{N1}^2}{c^2}\right)^{-1/2} \quad -(3)$$

using:

$$r_1 = \frac{r}{m(r)^{1/2}} \quad -(4)$$

and

$$v_{N1}^2 = \frac{v_N^2}{m(r)} \quad -(5)$$

$$-(6)$$

the hamiltonian becomes:

$$H = m(r)\left(m(r) - \frac{v_N^2}{m(r)c^2}\right)^{-1/2} - m(r)^{1/2}\frac{mMG}{r}$$

where the Newtonian momentum is:

$$\underline{p}_N = m\,\underline{v}_N. \quad -(7)$$

Therefore the hamiltonian is:

$$-(8)$$

$$H = \frac{m(r)}{\left(m(r) - \frac{p_N^2}{m(r)c^2}\right)^{1/2}} - m(r)^{1/2}\frac{e^2}{4\pi\epsilon_0 r}$$

For the H atom. Eq. (8) can be written as:

$$H = m(r)^{1/2}\left(1 - \frac{p_N^2}{m^2c^2 m(r)^{3/2}}\right)^{-1/2} - m(r)^{1/2}\frac{e^2}{4\pi\epsilon_0 r} \quad -(9)$$

If

$$p_N << mc \quad -(10)$$

$$H \sim m(r)^{1/2}\left(1 + \frac{p_N^2}{2m^2c^2 m(r)^{3/2}} + \dots\right)mc^2 - m(r)^{1/2}\frac{e^2}{4\pi\epsilon_0 r} \quad -(11)$$

Fm eq. (1) the rest energy is:

$$E_0 = m(r)^{1/2}mc^2 \quad -(12)$$

So

$$H_0 := H - m(r)^{1/2}mc^2$$

$$= \frac{1}{m(r)}\frac{p_N^2}{2m} - m(r)^{1/2}\frac{e^2}{4\pi\epsilon_0 r} \quad -(13)$$

Using

$$p_N^2\phi = -\hbar^2\nabla^2\phi \quad -(14)$$

the quantized energy levels of the H atom in the theory are given by:

$$H_0\phi = E\phi \quad -(15)$$

$$= -\frac{1}{m(r)}\frac{\nabla^2\phi}{2m} - m(r)^{1/2}\frac{e^2}{4\pi\epsilon_0 r}\phi$$

The expectation values of the energy levels are

3)

$$E = \int \psi^* E \psi \, d\tau \qquad -(16)$$

$$= -\frac{1}{2m}\int \psi^* \frac{\nabla^2}{m(r)}\psi \, d\tau - \frac{e^2}{4\pi\epsilon_0}\int \psi^* \frac{m(r)^{1/2}}{r}\psi \, d\tau$$

In the first approximation use the ψ functions of the H atom as in previous work:

$$\psi = R(r)\, Y(\theta, \phi) \qquad -(17)$$

where R is characterized by the quantum numbers n and l and where Y, the spherical harmonics, are characterized by the quantum numbers l and

$$m = -l, \ldots, l \qquad -(18)$$

Here R is the radial wave function and Y is the angular part of ψ.

In the non-relativistic H atom:

$$E = -\frac{m e^4}{32\pi^2 \epsilon_0^2 \hbar^2 n^2} \qquad -(18)$$

where

$$n = 1, 2, \ldots \qquad -(19)$$

the principal quantum number. So from eq. (18), E is independent of l. The result (18) is obtained from eq. (16) with:

$$m(r) = 1 \qquad -(20)$$

i.e.

$$E = -\frac{1}{2m}\int \psi^* \nabla^2 \psi \, d\tau - \frac{e^2}{4\pi\epsilon_0}\int \psi^* \frac{1}{r}\psi \, d\tau$$

$$-(2$$

4) Note carefully that each energy level of the H atom is shifted to a different extent by the $m(r)$ function of the most general spherically symmetric spacetime. Eq. (16) is an approximation to the relativistic structure of the H atom, Eq. (21) is the non-relativistic H atom. It might be that the $m(r)$ function splits the energy levels of the H atom, and may account for the Lamb shift.

The starting hamiltonian can also be expressed as:

$$H = E + U = \left(m(r_1)\left(p_1^2 c^2 + m^2 c^4\right)\right)^{1/2} - \frac{e^2}{4\pi\epsilon_0 r_1} \quad -(22)$$

and this is a generalization to m space of Sommerfeld and Dirac theory.
