427(1): Hamilton Equations for m Theory

Consider the relativistic momentum in m theory:

$$p_1 = \gamma m v_1 \quad -(1)$$

in frame (r_1, ϕ), where:

$$r_1 = \frac{r}{m(r_1)^{1/2}} \quad -(2)$$

Here:

$$v_1 = \frac{v_N}{m(r_1)^{1/2}} \quad -(3)$$

where v_N is the Newtonian velocity.

It follows that:

$$p_1^2 c^2 = \gamma^2 m^2 v_1^2 c^2 \quad -(4)$$

where

$$\gamma = \left(m(r_1) - \frac{v_1^2}{c^2} \right)^{-1/2} \quad -(5).$$

Therefore:

$$p_1^2 c^2 = \gamma^2 m^2 c^4 \frac{v_1^2}{c^2} \quad -(6)$$

$$= \gamma^2 m^2 c^4 \left(m(r_1) - \frac{1}{\gamma^2} \right)$$

$$= m(r_1) \gamma^2 m^2 c^4 - m^2 c^4$$

$$= \frac{E^2}{m(r_1)} - m^2 c^4$$

It follows that:

$$E^2 = m(r_1)\left(p_1^2 c^2 + m^2 c^4\right) \quad -(7)$$

as in UFT424.

The hamiltonian of m theory is therefore:

2)

$$H = m(r_1)\gamma mc^2 - \frac{mMG}{r_1} \quad -(8)$$

$$= \left(m(r_1)\left(p_1^2c^2 + m^2c^4\right)\right)^{1/2} - \frac{mMG}{r_1}$$

Now define the canonically conjugate generalized coordinates:

$$p_r = p_1, \quad q_r = r_1 \quad -(9)$$

and

$$p_\phi = L_1, \quad q_\phi = \phi \quad -(10)$$

The Hamilton equations are:

$$\dot{p}_r = -\frac{\partial H}{\partial q_r} \quad -(11)$$

$$\dot{q}_r = \frac{\partial H}{\partial p_r} \quad -(12)$$

$$\dot{p}_\phi = -\frac{\partial H}{\partial q_\phi} \quad -(13)$$

$$\dot{q}_\phi = \frac{\partial H}{\partial p_\phi} \quad -(14).$$

From eq. (11):

$$\dot{p}_1 = -\frac{mMG}{r_1^2} - \frac{\partial}{\partial r_1}\left(m(r_1)\left(p_1^2c^2+m^2c^4\right)\right)^{1/2}$$

$$= -\frac{mMG}{r_1^2} - \frac{E}{m(r_1)^{1/2}}\frac{\partial}{\partial r_1}\left(m^{1/2}(r_1)\right) \quad -(15)$$

3) From the Lagrangian analysis of UFT417:

$$\dot{p}_1 = -\frac{mMG}{r_1^2} - \frac{mc^2\gamma}{2}\frac{\partial m(r_1)}{\partial r_1} \quad -(16)$$

Now use:

$$E = m(r_1)\gamma mc^2 \quad -(17)$$

and compare eqs. (15) and (16):

$$\frac{E}{m(r_1)^{1/2}}\frac{\partial}{\partial r_1}\left(m^{1/2}(r_1)\right) = \frac{E}{2m(r_1)}\frac{\partial m(r_1)}{\partial r_1} \quad -(18)$$

i.e.

$$\frac{\partial}{\partial r_1}\left(m^{1/2}(r_1)\right) = \frac{1}{2m^{1/2}(r_1)}\frac{\partial m(r_1)}{\partial r_1}. \quad -(19)$$

Let

$$f = m(r_1)^{1/2} \quad -(20)$$

then

$$\frac{\partial f}{\partial r_1} = \frac{\partial f}{m(r_1)}\frac{\partial m(r_1)}{\partial r_1} \quad -(21)$$

$$= \frac{1}{2m^{1/2}(r_1)}\frac{\partial m(r_1)}{\partial r_1}$$

Therefore eq. (21) is true, Q.E.D.

The Hamilton and Euler Lagrange equations produce the same force due to m space:

$$\boxed{F = -\frac{mc^2\gamma}{2}\frac{\partial m(r_1)}{\partial r_1} = -\frac{E}{m(r_1)^{1/2}}\frac{\partial}{\partial r_1}\left(m^{1/2}(r_1)\right) \quad -(22)}$$

4) The Hamilton equation (12) gives:

$$\dot{r}_1 = \frac{\partial H}{\partial p_1} \quad -(23) \qquad -(24)$$

where

$$H = \left(n(r_1)\left(p_1^2 c^2 + m^2 c^4 \right) \right)^{1/2} - \frac{mMG}{r_1}$$

Therefore:

$$\frac{\partial H}{\partial p_1} = \frac{n(r_1) p_1 c^2}{n(r_1) \gamma m c^2} = \frac{p_1}{m\gamma} \quad -(25)$$

where we have used:

$$E = n(r_1)\gamma m c^2 = \left(n(r_1)\left(p_1^2 c^2 + m^2 c^4 \right) \right)^{1/2} \quad -(26)$$

So

$$p_1 = m\gamma \dot{r}_1 \quad -(27)$$

Q.E.D.

To extend to plane polar coordinates use:

$$\underline{p}_1 = \gamma m \dot{\underline{r}}_1 = \gamma m \left(\dot{r}_1 \underline{e}_r + r_1 \dot{\phi} \underline{e}_\phi \right) \quad -(28)$$

So

$$p_1^2 = p_r^2 + \frac{p_\phi^2}{r^2} \quad -(29)$$

$$= p_r^2 + \frac{L^2}{r^2}$$

Eq. (13) gives:

$$\dot{L} = \frac{dL}{dt} = 0 \quad -(30)$$

so L is a constant of motion.

Eq. (14) gives:

$$\dot{\phi} = \frac{\partial H}{\partial L_1} \quad -(31)$$

where

$$H = \left(m(r_1) \left(\left(p_r^2 + \frac{L^2}{r^2} \right) c^2 + m^2 c^4 \right) \right)^{1/2} - \frac{mMG}{r_1}.$$

It follows that:

$$\dot{\phi} = \frac{\partial H}{\partial L} = \frac{m(r_1) L_1 c^2}{r_1^2 m(r_1) \gamma m c^2} \quad -(32) \quad -(33)$$

i.e.

$$L_1 = \gamma m r_1^2 \dot{\phi} \quad -(34).$$

Q.E.D.

The m theory in Hamilton and Euler-Lagrange formalism is rigorously self consistent, both formalisms give the same results.

The Hamilton Jacobi equations are found from eq. (32) using:

$$p_r = \frac{\partial S}{\partial r}, \quad p_\phi = \frac{\partial S}{\partial \phi} \quad -(35)$$

where S is the action:

$$S = S_r + S_\phi \quad -(36).$$

Therefore the Hamilton Jacobi equation of m theory

b) are:

$$E_1 = \left(n(r_1) \left(\left(\left(\frac{\partial S_r}{\partial r} \right)^2 + \frac{L_1^2}{r^2} \right) c^2 + m^2 c^4 \right) \right)^{1/2} - \frac{n \hbar c}{r_1} \quad -(37)$$

and

$$L_1 = \frac{\partial S_\phi}{\partial \phi} = \text{constant} \quad -(38)$$

These equations can be integrated by computer algebra to give S_r and S_ϕ. This is a route to quantization because $\hbar$ is the quantum of action. The spin connection can be found from eq. (22)