426(6): Hamilton Jacobi Equations for Special Relativity

Define the hamiltonian as:

$$H = (p^2c^2 + m^2c^4)^{1/2} - \frac{mMG}{r} \quad -(1)$$

So the canonically conjugate variables are:

$$p = \gamma m v_N, \quad q = r. \quad -(2)$$

The first Hamilton equation gives:

$$\dot{p} = -\frac{\partial H}{\partial r} = -\frac{mMG}{r^2} \quad -(3)$$

i.e.

$$\frac{d}{dt}(\gamma m v_N) = -\frac{mMG}{r^2} \quad -(4)$$

This is the same result as from previous UFT papers on the Euler Lagrange method in special relativity.

The second Hamilton equation gives:

$$\dot{r} = \frac{\partial H}{\partial p} = \frac{pc^2}{(p^2c^2 + m^2c^4)^{1/2}}$$

$$= \frac{pc^2}{\gamma mc^2} = \frac{p}{\gamma m} \quad -(5)$$

so

$$p = \gamma m \dot{r} \quad -(6)$$

In the above analysis:

$$v_N = \dot{r} \quad -(7)$$

To extend to plane polar coordinates use:

$$\underline{p} = \gamma m \dot{\underline{r}} = \gamma m (\dot{r}\,\underline{e}_r + r\dot{\phi}\,\underline{e}_\phi) \quad -(8)$$

So

$$p^2 = p_r^2 + \frac{p_\phi^2}{r^2} \quad -(9)$$

2) where
$$P_\phi = L, \quad q_\phi = \phi \quad -(10)$$
Here
$$L = \gamma m r^2 \dot{\phi} \quad -(11)$$
is the angular momentum. The first Hamilton equation gives:
$$\dot{L} = -\frac{\partial H}{\partial \phi} = 0 \quad -(12)$$
so L is a constant of motion.
The second Hamilton equation gives:
$$\dot{q} = \frac{\partial H}{\partial L} \quad -(13)$$
where
$$H = \left(p_r^2 c^2 + \frac{L^2}{r^2} c^2\right)^{1/2} - \frac{mMG}{r} \quad -(14)$$
so
$$\dot{q}_\phi = \frac{Lc^2}{r^2 \gamma m c^2} = \frac{L}{m\gamma r^2} \quad -(15)$$
so:
$$L = \gamma m r^2 \dot{q}_\phi = \gamma m r^2 \dot{\phi} \quad -(16)$$
This is the same result as from the Lagrangian analysis Q.E.D.
The Hamilton Jacobi equation is found using:
$$p_r = \frac{\partial S}{\partial r}, \quad p_\phi = \frac{\partial S}{\partial \phi} \quad -(17)$$
where S is the action:
$$S = S_r + S_\phi \quad -(18)$$
Therefore the two Hamilton Jacobi equations are:
$$E = \left(c^2\left(\left(\frac{\partial S_r}{\partial r}\right)^2 + \frac{L^2}{r^2}\right) + m^2 c^4\right)^{1/2} - \frac{mMG}{r} \quad -(19)$$
and
$$L = \frac{\partial S_\phi}{\partial \phi} = \text{constant} \quad -(20)$$

3) The two Evans Eckardt equations are:

$$\frac{dE}{dt} = \frac{dH}{dt} = 0 \quad -(21)$$

and

$$\frac{dL}{dt} = 0 \quad -(22)$$

So eqns. (19) to (22) can be considered to give new physics. For example eq. (19) can be considered to give new physics with eq. (21), and eqn. (19) can be integrated numerically to give the action S_r. This paves the way for quantization because the quantum of action is $\hbar$.