426(5): New Formulation of the Hamilton Equations in Special Relativity

Begin by summarizing the equations of m theory:

$$\mathcal{L} = p_1 \dot{q}_1 - H \quad -(1)$$

$$p_1 \dot{q}_1 = m(r_1)\gamma m c^2 - \frac{mc^2}{\gamma} \quad -(2)$$

$$= \frac{p_1^2 c^2}{\gamma m c^2} - H$$

From fundamentals:

$$\dot{p}_1 = \frac{\partial \mathcal{L}}{\partial r_1} = -\frac{\partial H}{\partial r_1} \quad -(3)$$

and

$$\frac{\partial}{\partial r_1}\left(m(r_1)\gamma m c^2 - \frac{mc^2}{\gamma} \right) = 0 \quad -(4)$$

Here:

$$H = \gamma m(r_1) m c^2 - \frac{mMG}{r_1} \quad -(5)$$

and

$$\mathcal{L} = -\frac{mc^2}{\gamma} + \frac{mMG}{r_1} \quad -(6)$$

where

$$\gamma = \left(m(r_1) - \frac{v_1^2}{c^2} \right)^{-1/2} \quad -(7)$$

Using:

$$v_1^2 = \dot{r}_1^2 + r_1^2 \dot{\phi}^2 \quad -(8)$$

it follows as in UFT425 ~~that~~:

$$\frac{dm(r_1)}{dr_1} = -\frac{2L^2}{\gamma^2 c^2 m^2 r_1^3}\left(\frac{1 - \gamma^2 m(r_1)}{1 + \gamma^2 m(r_1)} \right) \quad -(9)$$

where:

2)

$$L = \gamma r_1^2 m \dot{\phi} \quad -(10)$$

is the conserved angular momentum of n theory. Here:

$$r_1 = \frac{r}{n(r)^{1/2}} \quad -(11)$$

In the limit of special relativity:

$$n(r_1) \rightarrow 1, \quad \frac{dn(r_1)}{dr_1} \rightarrow 0 \quad -(12)$$

and

$$\gamma \rightarrow \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} \quad -(13)$$

where the Newtonian velocity is defined by:

$$v_N^2 = \dot{r}^2 + r^2 \dot{\phi}^2 \quad -(14)$$

in frame (r, ϕ).

The flat space of special relativity is approached as:

$$r_1^3 \rightarrow \infty \quad -(15)$$

so in eq. (9):

$$\frac{dn(r_1)}{dr_1} \rightarrow 0. \quad -(16)$$

In the limit of S.R. eq. (4) becomes:

$$\frac{\partial \gamma}{\partial r} = \frac{\partial}{\partial r}\left(\frac{1}{\gamma}\right). \quad -(17)$$

This is true if

$$\frac{\partial \gamma}{\partial r} = \frac{\partial}{\partial r}\left(\frac{1}{\gamma}\right) = 0 \quad -(18)$$

i.e.

$$\boxed{\frac{\partial v_N}{\partial r} = 0} \quad -(19)$$

3) Eq. (19) follows immediately in Hamiltonian dynamics if the canonically conjugate generalized coordinates are defined as:

$$p = m v_N \, ; \quad q = r \quad -(20)$$

The canonically conjugate variable are independent so:

$$\frac{\partial p}{\partial q} = 0 \quad -(21)$$

which is eq. (19). The latter means that:

$$\boxed{\frac{\partial}{\partial r}\left(\dot{r}^2 + r^2\dot{\phi}^2\right) = 0} \quad -(22)$$

i.e.

$$\boxed{\frac{\partial}{\partial t}\left(\dot{r}^2 + r^2\dot{\phi}^2\right) = 0} \quad -(23)$$

using

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial t}\frac{dt}{dr} \quad -(24)$$

Eq. (23) contains the Evans Eckardt equations for special relativity:

$$\frac{dH}{dt} = 0 \quad -(25)$$

and

$$\frac{dL}{dt} = 0 \quad -(26)$$

where

$$H = \gamma m c^2 - \frac{mMG}{r} \quad -(27)$$

$$L = \gamma m r^2 \dot{\phi} \quad -(28)$$

Eqs. (23), (27) and (28) must be solved simultaneously

4) Eqs. (22) and (27) give:

$$\frac{\partial H}{\partial r} = \frac{mMG}{r^2} \quad -(29)$$

So

$$\dot{p} = -\frac{\partial H}{\partial r} = -\frac{mMG}{r^2} \quad -(30)$$

Eqs. (23) and (28) give:

$$\frac{dL}{dt} = \gamma \frac{d}{dt}\left(mr^2\dot{\phi}\right) = 0 \quad -(31)$$

because

$$\frac{d\gamma}{dt} = 0 \quad -(32)$$

With the choice (28):

$$\boxed{H = mc^2\left(1 - \frac{p^2}{m^2c^2}\right)^{-1/2} - \frac{mMG}{r}} \quad -(33)$$

where

$$\boxed{p = mv_N} \quad -(34)$$

So

$$\dot{r} = \frac{\partial H}{\partial p} = \gamma^3 \frac{p}{m} = \gamma^3 v_N \quad -(35)$$

It follows that:

$$m\ddot{r} = m\gamma^3 \dot{v}_N = -\frac{mMG}{r^2} \quad -(36)$$

Finally use:

$$\boxed{\frac{d}{dt}\left(\gamma m v_N\right) = m\gamma^3 \dot{v}_N = -\frac{mMG}{r^2}} \quad -(37)$$

5) Eq. (37) is the equation of special relativity used in previous work to produce a precessing ellipse.

The choice (20) ensures that:

$$\frac{\partial p}{\partial q} = 0 \quad -(38)$$

and

$$\frac{\partial V_N}{\partial r} = 0 \quad -(39)$$

The Hamiltonian equations (22) and (23) give new dynamics, and can be integrated numerically, and both the Lagrangian and Hamiltonian methods give eq. (37).

Using the choice:

$$p_\phi = L, \; r_\phi = \phi \quad -(40)$$

we have:

$$\dot{L} = \dot{p}_\phi = -\frac{\partial H}{\partial \phi} = 0 \quad -(41)$$

So the angular momentum is a constant of motion. The relativistic momentum for eq. (37) is:

$$\underline{p} = \gamma m \underline{v}_N \quad -(42)$$

So the magnitude of $\underline{L}$ is:

$$\boxed{|\underline{L}| = |\underline{r} \times \underline{p}| = \gamma m r^2 \dot{\phi}} \quad -(43)$$

Using:

6)

$$\underline{v}_N = \dot{r}\,\underline{e}_r + r\dot{\phi}\,\underline{e}_\phi \quad -(44)$$

and

$$\underline{r} = \dot{r}\,\underline{e}_r \quad -(45)$$

In the Hamilton Jacobi formalism:

$$L = \frac{\partial S_\phi}{\partial \phi} = \text{constant} \quad -(46)$$

so

$$\boxed{\frac{\partial S_\phi}{\partial \phi} = \gamma m r^2 \dot{\phi}} \quad -(47)$$

and the action S_ϕ can be found by integration. Similarly the action S_r can be found as in the next note.