426(2): Development of the New Equation of Motion

The new equation is:

$$\frac{\partial \mathcal{L}}{\partial p} = p \frac{\partial \dot{q}}{\partial p} \quad -(1)$$

where p and q are canonically conjugate generalized coordinates. First note that:

$$\frac{\partial \mathcal{L}}{\partial p} = \frac{\partial \mathcal{L}}{\partial \dot{q}} \frac{\partial \dot{q}}{\partial p} \quad -(2)$$

so:

$$p = \frac{\partial \mathcal{L}}{\partial \dot{q}} \quad -(3)$$

and from the Euler Lagrange equations:

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial q} \quad -(4)$$

On the Newtonian level in frame (r, ϕ):

$$p_r = p, \quad q_r = r \quad -(5)$$

$$p_\phi = L, \quad q_\phi = \phi \quad -(6)$$

So eq. (1) gives:

$$\frac{\partial \mathcal{L}}{\partial p} = p \frac{\partial \dot{r}}{\partial p} \quad -(7)$$

and

$$\frac{\partial \mathcal{L}}{\partial L} = L \frac{\partial \dot{\phi}}{\partial p} \quad -(8)$$

The hamiltonian and lagrangian are:

$$H = \frac{1}{2m} \left(p_r^2 + p_\phi^2 \right) - \frac{mMG}{r} \quad -(9)$$

$$\mathcal{L} = \frac{1}{2m} \left(p_r^2 + p_\phi^2 \right) + \frac{mMG}{r} \quad -(10)$$

The canonical Hamilton equations are:

$$\dot{p} = -\frac{\partial H}{\partial r}, \quad \dot{r} = \frac{\partial H}{\partial p} \quad -(11)$$

These may be developed into the vector Hamilton equations:

$$\dot{\underline{p}} = -\frac{\partial H}{\partial \underline{r}} = -\underline{\nabla} H \quad -(12)$$

and

$$\dot{\underline{r}} = \frac{\partial H}{\partial \underline{p}} = \frac{\partial H}{\partial p_r}\underline{e}_r + \frac{\partial H}{\partial p_\phi}\underline{e}_\phi \quad -(13)$$

Eq. (12) is:

$$\underline{p} = m\left(\dot{r}\,\underline{e}_r + r\dot{\phi}\,\underline{e}_\phi\right) \quad -(14)$$

$$= \frac{\partial H}{\partial \dot{r}}\underline{e}_r + \frac{\partial H}{r\dot{\phi}}\underline{e}_\phi$$

It follows that:

$$\dot{p}_r = m\dot{r} = -\frac{\partial H}{\partial r} \quad -(15)$$

and

$$\dot{p}_\phi = mr\dot{\phi} = -\frac{\partial H}{\partial r\dot{\phi}} \quad -(16)$$

The Newtonian velocity is:

$$\underline{v}_N = \dot{r}\,\underline{e}_r + r\dot{\phi}\,\underline{e}_\phi \quad -(17)$$

so

$$v_N^2 = \dot{r}^2 + r^2\dot{\phi}^2 \quad -(18)$$

and the hamiltonian may be written as: $-(19)$

$$H = \frac{1}{2m}\underline{p}\cdot\underline{p} - \frac{mMG}{r} = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\phi}^2\right) - \frac{mMG}{r}$$

Therefore the Hamilton equations can be interpreted as:

$$\dot{\underline{p}} = m\dot{\underline{v}}_N = -\frac{\partial H}{\partial r}\underline{e}_r \quad -(20)$$

i.e.

$$\dot{\underline{p}} = m\frac{\partial \underline{v}_N}{\partial t} = -\frac{mMG}{r^2}\underline{e}_r \quad -(21)$$

and

$$\underline{v}_N = \dot{\underline{r}} = \frac{\partial H}{\partial \underline{p}} = \dot{r}\,\underline{e}_r + r\dot{\phi}\,\underline{e}_\phi \quad -(22)$$

Eq. (22) splits into the usual component Hamilton equations:

$$\dot{r} = \frac{\partial H}{\partial p} \quad -(23)$$

and

$$r\dot{\phi} = \frac{\partial H}{\partial (r\dot{\phi})} \quad -(24)$$

It is therefore possible to choose:

$$p_r = mv_N \quad -(25)$$

and

$$q_r = r \quad -(26)$$

and the Hamilton equation:

$$\dot{r} = \frac{\partial H}{\partial p} \quad -(27)$$

with Hamiltonian:

$$H = \frac{p^2}{2m} - \frac{mMG}{r} \quad -(28)$$

gives

$$\dot{r} = v_N = \frac{p}{m} \quad -(29)$$

Q.E.D.

With these preliminaries eq. (1) can be written as:

$$\frac{\partial \mathcal{L}}{\partial p} = \frac{\partial \mathcal{L}}{\partial v_N}\frac{\partial v_N}{\partial p} = p\frac{\partial \dot{q}}{\partial p} = p\frac{\partial \dot{q}}{\partial v_N}\frac{\partial v_N}{\partial p} \quad -(30)$$

where

$$v_N = \dot{q} \quad -(31)$$

so

$$\frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial v_N} = p \quad -(32)$$

, which is a Lagrange equation.

In m theory

$$m\gamma\left(v_{1N} - \frac{1}{2}c^2\frac{dm(r_1)}{dv_{1N}}\right) = \gamma m v_{1N} \quad -(32)$$

so

$$\boxed{\frac{dm(r_1)}{} = 0} \quad -(33)$$