426(1): Other Fundamental Equations of the Euler Lagrange Hamilton Dynamics

The use of these equations will introduce further information into the theory. For example the angular momentum is defined by:

$$L = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} \quad -(1)$$

where $\mathcal{L}$ is the lagrangian:

$$\mathcal{L} = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\phi}^2 \right) + \frac{mMG}{r} \quad -(2)$$

on the Newtonian level. Using the Euler Lagrange equations:

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \frac{\partial \mathcal{L}}{\partial \phi} \quad -(3)$$

it follows that

$$\dot{L} = \frac{dL}{dt} = 0 \quad -(4)$$

In Hamilton dynamics:

$$\dot{L} = \dot{p}_\phi = -\frac{\partial H}{\partial \phi} = 0 \quad -(5)$$

where the hamiltonian is:

$$H = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\phi}^2 \right) - \frac{mMG}{r} \quad -(6)$$

so

$$\dot{L} = \frac{\partial \mathcal{L}}{\partial \phi} = -\frac{\partial H}{\partial \phi} = 0 \quad -(7)$$

and

$$\dot{\phi} = \frac{\partial H}{\partial L} \quad -(8)$$

The first Evans Eckardt equation is defined in Euler / Lagrange / Hamilton dynamics as:

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} = -\frac{\partial \mathcal{L}}{\partial t} = 0 \quad -(9)$$

The use of the Hamilton dynamics shows that the Evans
Eckardt equations are:

$$\boxed{\frac{dH}{dt} = -\frac{\partial \mathcal{L}}{\partial t} = 0} \quad -(10)$$

and

$$\boxed{\frac{dL}{dt} = -\frac{\partial H}{\partial \phi} = 0} \quad -(11)$$

where

$$\dot{\phi} = \frac{\partial H}{\partial L} \quad -(12)$$

Eqs. (10) to (12) are general, and can be used in
theory.

On the Newtonian level:

$$L = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = mr^2 \dot{\phi} \quad -(13)$$

and the hamiltonian is:

$$H = \frac{1}{2} m \left(\dot{r}^2 + \frac{L^2}{m^2 r^2} \right) \quad -(14)$$

so

$$\frac{\partial H}{\partial L} = \frac{mL}{m^2 r^2} = \dot{\phi} \quad -(15)$$

Q.E.D. In special relativity the Hamilton canonical
variables are:

$$p_r = \gamma m \dot{r}, \quad \dot{q}_r = \dot{r}, \quad -(16)$$

$$p_\phi = \gamma m r \dot{\phi}, \quad \dot{q}_\phi = r \dot{\phi} \quad -(17)$$

and

$$\underline{p} = p_r \, \underline{e}_r + p_\phi \, \underline{e}_\phi \quad -(18)$$

The Hamilton equations are:

$$\dot{p} = -\frac{\partial H}{\partial q} \quad -(19)$$

and

$$\dot{q} = \frac{\partial H}{\partial p} \quad -(20)$$

where

$$H = p\dot{q} - \mathcal{L} \quad -(21)$$

in generalized conjugate coordinates p and q.

Therefore:

$$\dot{r} = \frac{\partial H}{\partial p_r} \quad -(22)$$

and

$$r\dot{\phi} = \frac{\partial H}{\partial p_\phi} \quad -(23)$$

in the plane polar system (r, ϕ). In this system:

$$\underline{p} = \gamma m \underline{v}_N \quad -(24)$$

where

$$\underline{v}_N = \dot{r}\,\underline{e}_r + r\dot{\phi}\,\underline{e}_\phi \quad -(25)$$

and

$$v_N^2 = v_r^2 + v_\phi^2 \quad -(26)$$
$$= \dot{r}^2 + r^2\dot{\phi}^2$$

The Hamiltonian of special relativity is:

$$H = \gamma m c^2 - \frac{mMG}{r} \quad -(27)$$

where

$$\gamma = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} \quad -(28)$$

It follows that:

$$\dot{q} = \frac{\partial H}{\partial p} = \frac{\partial H}{\partial v_N}\frac{\partial v_N}{\partial p} \quad -(29)$$

4) where
$$\underline{p} = m\underline{v}_N = p_r \underline{e}_r + p_\phi \underline{e}_\phi \quad -(30)$$

so
$$v_N^2 = \frac{p^2}{m^2} = \frac{1}{m^2}\left(p_r^2 + p_\phi^2\right) \quad -(31)$$

and
$$p = |\underline{p}| = \gamma m v_N \quad -(32)$$
$$= m v_N \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2}$$

It follows that:
$$\frac{\partial p}{\partial v_N} = m\left(\gamma + v_N \frac{d}{dv_N}\left(1 - \frac{v_N^2}{c^2}\right)^{-1/2}\right)$$
$$= m\left(\gamma + \gamma^3 \frac{v_N^2}{c^2}\right)$$
$$= \gamma^3 m \left(\frac{1}{\gamma^2} + \frac{v_N^2}{c^2}\right) \quad -(33)$$
$$= m\gamma^3$$

So the Hamilton equation (20) produces:
$$\dot{q} = \frac{1}{m\gamma^3} \frac{\partial H}{\partial v_N} \quad -(34)$$

where
$$\frac{\partial H}{\partial v_N} = m\gamma^3 v_N \quad -(35)$$

so
$$\boxed{\dot{q} = v_N} \quad -(36)$$

Q.E.D.

It follows that:
$$\underline{\dot{q}} = \dot{q}_r \underline{e}_r + \dot{q}_\phi \underline{e}_\phi \quad -(37)$$
$$= \dot{r}\, \underline{e}_r + r\dot{\phi}\, \underline{e}_\phi$$

so

$$\dot{q}_r = \dot{r} \quad -(38)$$

and

$$\dot{q}_\phi = r\dot{\phi} \quad -(39)$$

Q.E.D.

Having defined the mathematics at the Newtonian and special relativistic levels, it is now possible to produce extra information in the theory. This information is missing from a pure Lagrangian analysis but provides many new insights.

For example:

$$H = p\dot{q} - \mathcal{L} \quad -(40)$$

can be used with:

$$\frac{\partial H}{\partial p} = \dot{q} \quad -(41)$$

and

$$\frac{\partial H}{\partial q} = -\dot{p} \quad -(42)$$

Eqs. (40) and (42) produce:

$$\dot{p} = \frac{\partial H}{\partial q} = \frac{\partial (p\dot{q})}{\partial q} - \frac{\partial \mathcal{L}}{\partial q} \quad -(43)$$

and this equation has already been used in UFT425 to produce a new equation for $dm(r_1)/dr_1$.

Eqs. (40) and (41) produce:

$$\dot{q} = \frac{\partial H}{\partial p} = \frac{\partial (p\dot{q})}{\partial p} - \frac{\partial \mathcal{L}}{\partial p} \quad -(44)$$

and eq. (44) produces new information in the theory.

Eq. (44) is:

$$\dot{q} = \frac{\partial H}{\partial p} = \dot{q} + p\frac{\partial \dot{q}}{\partial p} - \frac{\partial \mathcal{L}}{\partial p} \quad -(45)$$

Using eq. (41):

$$\boxed{\frac{\partial \mathcal{L}}{\partial p} = p\frac{\partial \dot{q}}{\partial p}} \quad -(46)$$

This is a new equation of motion of general use in classical dynamics. It can be used to find new information about the theory.

Eq. (46) can be tested as follows. On the Newtonian level:

$$\mathcal{L} = \frac{p^2}{2m} + \frac{mMG}{r} \quad -(47)$$

so

$$\frac{\partial \mathcal{L}}{\partial p} = v_N \quad -(48)$$

We have:

$$p = mv_N, \quad \dot{q} = v_N \quad -(49)$$

so

$$\frac{\partial p}{\partial \dot{q}} = m \quad -(50)$$

It follows that:

$$p\frac{\partial \dot{q}}{\partial p} = v_N \quad -(51)$$

so eq. (46) is true on the Newtonian level, Q.E.D.

In special relativity:

$$\mathcal{L} = -mc^2\left(1 - \frac{p^2}{m^2c^2}\right)^{1/2} + \frac{mMG}{r} \quad -(52)$$

so

$$\frac{\partial \mathcal{L}}{\partial p} = \frac{\partial \mathcal{L}}{\partial v_N}\frac{\partial v_N}{\partial p} = \frac{1}{m\gamma^3}\frac{\partial \mathcal{L}}{\partial v_N} \quad -(53)$$

and

$$\frac{\partial \mathcal{L}}{\partial v_N} = \gamma m v_N = p \quad -(54)$$

so

$$\frac{\partial \mathcal{L}}{\partial p} = \frac{v_N}{\gamma^2} \quad -(55)$$

Similarly:

$$\frac{\partial \dot{q}}{\partial p} = \frac{\partial \dot{q}}{\partial v_N}\frac{\partial v_N}{\partial p} = \frac{1}{m\gamma^3} \quad -(56)$$

and

$$p = \gamma m v_N \quad -(57)$$

So

$$p\frac{\partial \dot{q}}{\partial p} = \frac{v_N}{\gamma^2} = \frac{\partial \mathcal{L}}{\partial p} \quad -(58)$$

Q.E.D.

We are now ready to develop these equations in the theory and to develop a Hamilton Jacobi formalism of the theory.