

```
(%i1) kill(all);
(%o0) done
```

□ **1 Eq. (19)**

```
(%i1) E1: diff(W(x),x)=sqrt(2*m*alpha_x);
(%o1)  $\frac{d}{dx} W(x) = \sqrt{2} \sqrt{\alpha_x m}$ 
```

```
(%i2) ode2(E1,W(x),x);
(%o2)  $W(x) = \sqrt{2} \sqrt{\alpha_x m} x + \%c$ 
```

```
(%i3) E2: diff(W(x),x)=-sqrt(2*m*alpha_x);
(%o3)  $\frac{d}{dx} W(x) = -\sqrt{2} \sqrt{\alpha_x m}$ 
```

```
(%i4) E3: ode2(E1,W(x),x);
(%o4)  $W(x) = \sqrt{2} \sqrt{\alpha_x m} x + \%c$ 
```

□ **2 Eq. (20)**

```
(%i5) E4: 1/(2*m)*diff(W(z),z)^2+m*g*z=alpha_z;
(%o5)  $\frac{\left(\frac{d}{dz} W(z)\right)^2}{2m} + gmz = \alpha_z$ 
```

```
(%i6) ode2(E4,W(z),z);
(%t6)  $\frac{\left(\frac{d}{dz} W(z)\right)^2}{2m} + gmz = \alpha_z$ 
```

first order equation not linear in y'

```
(%o6) false
```

```
(%i7) E5: 1/(2*m)*diff(W(z),z)^2+m*g*z = alpha_z;
(%o7)  $\frac{\left(\frac{d}{dz} W(z)\right)^2}{2m} + gmz = \alpha_z$ 
```

(%i8) E6: solve(E5, diff(W(z), z));

(%o8) $\left[\frac{d}{dz} W(z) = -\sqrt{2} \sqrt{\alpha_z m - g m^2 z}, \frac{d}{dz} W(z) = \sqrt{2} \sqrt{\alpha_z m - g m^2 z} \right]$

(%i9) E7: diff(W(z), z) = sqrt(2*m*(-m*g*z+alpha_z));

(%o9) $\frac{d}{dz} W(z) = \sqrt{2} \sqrt{m(\alpha_z - g m z)}$

(%i10) E8: ode2(E7, W(z), z);

(%o10) $W(z) = \%c - \frac{2^{3/2} (\alpha_z m - g m^2 z)^{3/2}}{3 g m^2}$

(%i11) E9: diff(W(z), z) = -sqrt(2*m*(-m*g*z+alpha_z));

(%o11) $\frac{d}{dz} W(z) = -\sqrt{2} \sqrt{m(\alpha_z - g m z)}$

(%i12) E10: ode2(E9, W(z), z);

(%o12) $W(z) = \frac{2^{3/2} (\alpha_z m - g m^2 z)^{3/2}}{3 g m^2} + \%c$

□ 3 Eqs. (21, 22)

(%i13) S: W(x)+W(z)-E*t;

(%o13) $W(z) + W(x) - E t$

Insert solutions

(%i14) S: rhs(E3)+rhs(E10) -2*%c - E*t;

(%o14) $\frac{2^{3/2} (\alpha_z m - g m^2 z)^{3/2}}{3 g m^2} + \sqrt{2} \sqrt{\alpha_x m x} - E t$

(%i15) E11: beta_x = diff(S, alpha_x)+diff(S, E);

(%o15) $beta_x = \frac{m x}{\sqrt{2} \sqrt{\alpha_x m}} - t$

```
(%i16) E12: beta_z = diff(S, alpha_z) + diff(S, E);
```

$$(\%o16) \quad \beta_z = \frac{\sqrt{2} \sqrt{\alpha_z m - g m^2 z}}{g m} - t$$

□ **4 Eq. (23, 24)**

```
(%i23) expand(solve(E11, x));
```

$$(\%o23) \quad \left[x = \frac{\sqrt{2} \sqrt{\alpha_x m} t}{m} + \frac{\sqrt{2} \beta_x \sqrt{\alpha_x m}}{m} \right]$$

```
(%i22) expand(solve(E12, z));
```

Is $g m (t + \beta_z)$ positive, negative or zero?

$$(\%o22) \quad \left[z = -\frac{g t^2}{2} - \beta_z g t + \frac{\alpha_z}{g m} - \frac{\beta_z^2 g}{2} \right]$$