418(8): Self Consistency of the Potential Energy Calculation

The change in potential energy due to $m(r_1)$ is:

$$\Delta U = U_1 - U_2 = -\frac{mc^2}{2}\int_1^2 \gamma \frac{dm(r_1)}{dr_1}\, dr_1 \quad -(1)$$

$$= \int_1^2 F(vac)\, dr_1$$

The force $F(vac)$ is the force due to $m(r_1)$, and is traditionally known as the vacuum force. In this process, the work integral is the well known:

$$U_1 - U_2 = T_2 - T_1 = \int_1^2 F(vac)\, dr_1 \quad -(2)$$

and the total energy is conserved:

$$T_1 + U_1 = T_2 + U_2 \quad -(3)$$

In the limit:

$$\frac{dm(r_1)}{dr_1} = 0 \quad -(4)$$

it follows that:

$$\Delta U = 0. \quad -(5)$$

so there is no energy of this type in Minkowski spacetime, in which

$$m(r_1) = 1 \quad -(6)$$

The energy is derived from the general spherically symmetric spacetime.

From eq. (1):

$$\Delta U = U_1 - U_2 = -\frac{mc^2}{2}\int_1^2 \gamma\, dm(r_1) \quad -(7)$$

$$= -\frac{mc^2}{2}\frac{1}{\gamma}\Big|_1^2$$

In m theory the generalized Lorentz factor is:

$$\gamma = \left(n(r_1) - \frac{v_1^2}{c^2} \right)^{-1/2} \quad -(8)$$

Therefore: $$\int \gamma \, dn(r_1) = \frac{2}{\gamma} \quad -(9)$$

Note carefully that in eq. (7), if $n(r_1)$ has no dependence on r_1 then

$$dn(r_1) = 0, \quad \Delta U = 0. \quad -(10)$$

By definition:

$$\int_1^2 \gamma \frac{dn(r_1)}{dr_1} dr_1 = \int_1^2 \gamma f(r_1) \, dr_1 \quad -(11)$$

where

$$f(r_1) := \frac{dn(r_1)}{dr_1} \quad -(12)$$

Therefore

$$\int_1^2 \gamma \, dn(r_1) := \int_1^2 \gamma f(r_1) \, dr_1 \quad -(13)$$

Using integration by parts:

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx \quad -(14)$$

so

$$\int \gamma \frac{dn(r_1)}{dr_1} dr_1 = \gamma n(r_1) - \int n(r_1) \frac{d\gamma}{dr_1} dr_1$$

$$= \gamma n(r_1) - \int n(r_1) \, d\gamma \quad -(15)$$

If

$$n(r_1) = 1 \quad -(16)$$

it follows that:

$$\int \gamma \frac{dn(r_1)}{dr_1} dr_1 = 0 \quad -(17)$$

and

$$\gamma - \int d\gamma = 0 \quad -(18)$$

So eq. (15) is rigorously self consistent, Q.E.D.

Therefore:

$$\int n(r_1)\, d\gamma = \gamma n(r_1) - \int \gamma\, dn(r_1) \quad -(19)$$

and

$$\int \gamma\, dn(r_1) = \frac{2}{\gamma} \quad -(20)$$

so

$$\int n(r_1)\, d\gamma = \gamma n(r_1) - \frac{2}{\gamma} \quad -(21)$$

In the limit:

$$n(r_1) \rightarrow 1 \quad -(22)$$

it follows that:

$$\int \gamma\, dn(r_1) = 0 \quad -(23)$$

and

$$\int n(r_1)\, d\gamma = \gamma n(r_1) \quad -(24)$$

and using eq. (22):

$$\int d\gamma = \gamma \quad -(25)$$

Q.E.D. This is a rigorously self consistent result. The overall result is:

$$\Delta U = U_1 - U_2 = -\frac{mc^2}{2}\int_1^2 \gamma\, dn(r_1) = \int_1^2 F(vac)\, dr_1$$

$$= -mc^2\left(\left(n(r_1) - \frac{v_1^2}{c^2}\right)^{1/2} - n(r_1)^{1/2}\right) \quad -(26)$$

where

$$\frac{dn(r_1)}{dr_1} = 2\left(\frac{n(r)^{3/2}\, dn(r)/dr}{2n(r) - r\, dn(r)/dr}\right) \quad -(27)$$

and

$$\int \gamma \frac{dn(r_1)}{dr_1}\, dr_1 = \gamma n(r_1) - \int n(r_1)\, d\gamma \quad -(28)$$

4) The function $dm(r_1)/dr_1$ becomes infinite at:

$$\boxed{2m(r) = r\frac{dm(r)}{dr}} \quad -(29)$$

From eq. (28) this condition could be satisfied when:

$$r \to \infty \quad -(30)$$

i.e

$$\boxed{m(r_1) = \frac{v_1^2}{c^2}} \quad -(31)$$

In the (r, ϕ) coordinate system this means:

$$m^2(r) = \frac{v^2}{c^2} \quad -(32)$$

i.e

$$\boxed{m(r) = \frac{v}{c}} \quad -(33)$$

Under the condition (33):

$$\boxed{\Delta U \to \infty} \quad -(34)$$
