418(7): The Potential Energy due to m(r)

From previous notes this is defined as:

$$U_1 - U_2 = -\frac{mc^2}{2}\int_1^2 \gamma\, dm(r_1) \quad -(1)$$

$$= \int_1^2 F(vac)\, dr_1$$

where F(vac) is the force due to m(r₁), the vacuum force:

$$F(vac) = -\frac{mc^2}{2}\gamma \frac{dm(r_1)}{dr_1} \quad -(2)$$

$$= \gamma mc^2 \left(\frac{m(r)^{3/2}\frac{dm(r)}{dr}}{r\frac{dm(r)}{dr} - 2m(r)} \right)$$

Under the condition:

$$r\frac{dm(r)}{dr} = 2m(r) \quad -(3)$$

the potential energy due to m(r) becomes infinite, or "resonates".

In eq. (1):

$$\gamma = \left(m(r) - \frac{v_1^2}{c^2}\right)^{-1/2} \quad -(4)$$

is the generalized Lorentz factor of m theory. So the change in potential energy from state 1 to 2 is:

$$U_1 - U_2 = -\frac{mc^2}{2}\int_1^2 \left(m(r_1) - \frac{v_1^2}{c^2}\right)^{-1/2} dm(r_1) \quad -(5)$$

Assume that:

$$\frac{dm(r_1)}{dv_1} = 0 \quad -(6)$$

as in the derivation of the relativistic kinetic energy of m theory in Note 418(4), and use:

$$\int (a+bx)^{-1/2}\,dx = \frac{2}{b}(a+bx)^{1/2} \quad -(7)$$

It follows that:

$$U_1 - U_2 = -mc^2 \left(n(r_1) - \frac{v_1^2}{c^2} \right)^{1/2} \Bigg|_1^2 \quad -(8)$$

$$= -\frac{mc^2}{\gamma} \Bigg|_1^2$$

It is assumed that the initial state is that of a particle at rest:

$$v_1 = 0 \quad -(9)$$

Therefore:

$$U_1 - U_2 = -mc^2 \left(\left(n(r_1) - \frac{v_1^2}{c^2} \right)^{1/2} - n(r_1)^{1/2} \right)$$

$$= -mc^2 n(r_1)^{1/2} \left(\left(1 - \frac{v_1^2}{n(r_1)c^2} \right)^{1/2} - 1 \right)$$

$$= \int_1^2 F(vac)\,dr_1 \quad -(10)$$

where $F(vac)$ is defined by Eq. (2).

The potential energy associated with a particle at rest is

$$(U_1 - U_2)_0 = (\Delta U)_0 = n(r_1)^{1/2} E_0 \quad -(11)$$

where

$$E_0 = mc^2 \quad -(12)$$

) This is the same as the rest energy derived in Note 18(4).

In the classical limit:

$$v_1 << c \quad -(13)$$

so:

$$\Delta U = -mc^2 n(r_1)^{1/2} \left(\left(1 - \frac{v_1^2}{n(r_1)c^2} \right)^{1/2} - 1 \right)$$

$$\sim \frac{1}{2} \frac{mv_1^2}{n(r_1)^{1/2}} \quad -(14)$$

$$= \frac{1}{2} \frac{mv^2}{n(r)^{3/2}}$$

In the limit of flat spacetime:

$$n(r_1) \rightarrow 1 \quad -(15)$$

it follows that

$$U_1 - U_2 = T_2 - T_1 = \frac{1}{2} mv^2 \quad -(16)$$

with

$$U_1 + T_1 = U_2 + T_2 \quad -(17)$$

If the initial condition is that of a particle at rest, then:

$$T_1 = 0 \quad -(18)$$

so

$$T_2 = \Delta U = \frac{1}{2} mv^2 \quad -(19)$$

The fully relativistic result is:

$$T_2 = \Delta U = -mc^2 n(r_1)^{1/2} \left(\left(1 - \frac{v_1^2}{n(r_1)c^2} \right)^{1/2} - 1 \right) \quad -(20)$$

Any function $n(r_1)$ imparts this energy to material matter

4) Note carefully that F(vac) is zero if:

$$\frac{dn(r_1)}{dr_1} = 0 \quad -(21)$$

so eqs. (19) and (20) must be interpreted as:

$$T_2 = \Delta U \neq 0 \quad -(22)$$

if and only if:

$$\frac{dn(r_1)}{dr_1} \neq 0 \quad -(23)$$

Therefore eq. (19) is the limit when:

$$\frac{dm(r_1)}{dr_1} \to 0 \; ; \; m(r_1) \to 1 \; ; \; v_1 \ll c \quad -(24)$$

but with $dm(r_1)/dr_1$ being identically non-zero the integral (1) can also be evaluated by defining:

$$f(r_1) = \frac{dm(r_1)}{dr_1} \quad -(25)$$

so

$$\Delta U = U_1 - U_2 = -\frac{mc^2}{2} \int_1^2 \gamma f(r_1)\, dr_1 \quad -(26)$$

This can be integrated by parts using:

$$\int u \left(\frac{dv}{dx}\right) dx = uv - \int v \frac{du}{dx}\, dx \quad -(27)$$

Therefore

$$\int \gamma f(r_1)\, dr_1 = \int \gamma \frac{dm(r_1)}{dr_1}\, dr_1 \quad -(28)$$

$$= \gamma m(r_1) - \int m(r_1) \frac{d\gamma}{dr_1}\, dr_1 \quad -(29)$$

i.e.

$$\int \gamma\, dm(r_1) = \gamma m(r_1) - \int m(r_1)\, d\gamma$$

so

$$\boxed{\Delta U = -\frac{mc^2}{2}\left(\gamma m(r_1) - \int m(r_1)\, d\gamma\right)} \quad -(30)$$

Eq. (30) makes it clear that if

$$n(r) = 1 \quad -(30)$$

then

$$\Delta U = -\frac{mc^2}{2}\left(\gamma - \int d\gamma\right) = 0 \quad -(31)$$

The formula for integration by parts is derived from:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \quad -(32)$$

So:

$$\int u\frac{dv}{dx}\,dx = \int \frac{d}{dx}(uv)\,dx - \int v\left(\frac{du}{dx}\right)dx \quad -(33)$$

When integrating from state 1 to state 2:

$$\int_1^2 u\frac{dv}{dx}\,dx = \int_1^2 \frac{d}{dx}(uv)\,dx - \int_1^2 v\left(\frac{du}{dx}\right)dx \quad -(34)$$

i.e.

$$\boxed{\int_1^2 u\frac{dv}{dx}\,dx = uv\Big|_1^2 - \int_1^2 v\left(\frac{du}{dx}\right)dx} \quad -(35)$$

where

$$uv\Big|_1^2 = (uv)_2 - (uv)_1 \quad -(36)$$

It follows that:

$$\boxed{\Delta U = -\frac{mc^2}{2}\left(\gamma n(r_1)\Big|_1^2 - \int_1^2 n(r_1)\,d\gamma\right)} \quad -(37)$$

This will be evaluated in the next note.