418(6): The Einstein Energy Equation in m Theory

Consider the relativistic linear momentum of m theory:

$$\underline{p}_1 = \gamma m \underline{v}_1 = \gamma m \underline{\dot{r}}_1 \quad -(1)$$

where

$$\gamma = \left(m(r) - \frac{v_1^2}{c^2} \right)^{-1/2} \quad -(2)$$

is the generalized Lorentz factor. It follows from eq. (1) that:

$$p_1^2 c^2 = \gamma^2 m^2 v_1^2 c^2 = \gamma^2 m^2 c^4 \frac{v_1^2}{c^2} \quad -(3)$$

From eq. (2):

$$\frac{1}{\gamma^2} = m(r)^2 - \frac{v_1^2}{c^2} \quad -(4)$$

so

$$\frac{v_1^2}{c^2} = m(r)^2 - \frac{1}{\gamma^2} \quad -(5)$$

From eqs. (3) and (5):

$$p_1^2 c^2 = \gamma^2 m^2 c^4 \left(m(r)^2 - \frac{1}{\gamma^2} \right) \quad -(6)$$

$$= m^2 c^4 \gamma^2 m(r)^2 - m^2 c^4$$

$$= E^2 - E_0^2$$

The total relativistic energy is:

$$E = m(r) \gamma m c^2 \quad -(7)$$

and the rest energy is

$$E_0 = m c^2 \quad -(8)$$

so

$$\boxed{E^2 = p_1^2 c^2 + E_0^2} \quad -(9)$$

This is the Einstein energy equation of m space.

2) Note carefully that eq. (9) is a generally covariant equation, whereas the Einstein energy equation in Minkowski space is Lorentz covariant:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(10)$$

In note 418(4) the relativistic kinetic energy in m space was calculated from the work integral of eq. (1) to be:

$$T = \gamma m(r_1) mc^2 - m(r_1)^{1/2} mc^2 \quad -(11)$$

$$= E - m(r_1)^{1/2} mc^2$$

For self consistency define:

$$E_0 = mc^2 \quad -(12)$$

So:

$$\boxed{T = E - m(r_1)^{1/2} E_0} \quad -(13)$$

Note carefully that this is also a generally covariant equation of m space.

In Minkowski or flat space:

$$T = E - E_0 \quad -(14)$$

Eq. (13) is rigorously consistent with eq. (32) of Note 417(7), which defined the reduced hamiltonian:

$$H_0 = H - m(r)^{1/2} mc^2 \quad -(15)$$

Q.E.D.