# General orbit theory in spherically symmetric spacetime

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## 3 Some results from Eqs. (42) and (43)

### 3.1 Euler-Lagrange equations

We first present the equations of motion based on m space in extension of the computer algebra work of UFT 415. The velocity of an orbiting object in observer space is

$$v = \dot{r}^2 + r^2\dot{\phi}^2 \tag{44}$$

and the radial coordinate and velocity in m space are

$$r_1 = \frac{r}{\sqrt{\mathrm{m}(r)}}, \tag{45}$$

$$v_1 = \frac{v}{\sqrt{\mathrm{m}(r)}}. \tag{46}$$

In addition, the time is transformed inversely to $r$:

$$t_1 = \sqrt{\mathrm{m}(r)}\, t. \tag{47}$$

As worked out in section 2, the potential energy is

$$E_{\mathrm{pot}} = -\sqrt{\mathrm{m}(r)}\,\frac{mMG}{r} \tag{48}$$

and the total relativistic energy is

$$E = (\mathrm{m}(r)\,\gamma - 1)\; mc^2 - \sqrt{\mathrm{m}(r)}\,\frac{mMG}{r} = \mathrm{const} \tag{49}$$

with the $\gamma$ factor of non-constant, spherically symmetric spacetime:

$$\gamma = \left(\mathrm{m}(r) - \frac{v_1^2}{c^2}\right)^{-1/2} = \left(\mathrm{m}(r) - \frac{\dot{r}^2 + r^2\dot{\phi}^2}{\mathrm{m}(r)\,c^2}\right)^{-1/2}. \tag{50}$$

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The conserverd angular momentum is

$$L = \gamma\, m\, r_1^2\, \dot\phi = \frac{\gamma}{\mathrm{m}(r)}\, m\, r^2\, \dot\phi = \mathrm{const.} \tag{51}$$

The equations of motion are derived as Euler-Lagrange equations from the relativistic Lagrangian

$$\mathscr{L} = -\frac{mc^2}{\gamma} + \sqrt{\mathrm{m}(r)}\, \frac{mMG}{r}. \tag{52}$$

The Euler-Lagrange equations in normalized form are obtained from the analytical calculation by computer algebra:

$$\boxed{\ddot\phi = \dot\phi\, \dot r \left( \frac{\frac{d}{dr}\,\mathrm{m}(r)}{\mathrm{m}(r)} \left( 2 - \frac{GM}{2\gamma\, c^2\, r\, \sqrt{\mathrm{m}(r)}} \right) + \frac{GM}{\gamma\, c^2\, r^2\, \sqrt{\mathrm{m}(r)}} - \frac{2}{r} \right),} \tag{53}$$

$$\boxed{\begin{aligned} \ddot r = & \left( \frac{d}{dr}\,\mathrm{m}(r) \right) \left( c^2\, \mathrm{m}(r) + \frac{GM}{2\gamma^3 r\, \sqrt{\mathrm{m}(r)}} - \frac{3c^2}{2\gamma^2} \right) \\ & - \frac{\frac{d}{dr}\,\mathrm{m}(r)}{\mathrm{m}(r)}\, \dot\phi^2 r^2 \left( 2 - \frac{GM}{2\gamma c^2 r\, \sqrt{\mathrm{m}(r)}} \right) - \frac{GM\, \dot\phi^2}{\gamma\, c^2\, \sqrt{\mathrm{m}(r)}} \\ & + \dot\phi^2 r - \frac{GM\, \sqrt{\mathrm{m}(r)}}{\gamma^3\, r^2}. \end{aligned}} \tag{54}$$

The m function is to be predefined as a parameter of calculation. The equations and their results are very similar to the temporary version provided in UFT 415. Numerical examples are discussed in the next subsection.

Instead of executing the calculations in observer space $(r, \phi)$, we can completely switch to the m space coordinates $(r_1, \phi)$. With (45-47), all $r$-dependent quantities are transformed to $r_1$-dependent quantities, giving

$$E_{\mathrm{pot}} = -\frac{mMG}{r_1} \tag{55}$$

$$E = (\mathrm{m}_1(r)\, \gamma - 1)\; mc^2 - \frac{mMG}{r_1} \tag{56}$$

$$\gamma = \left( \mathrm{m}(r) - \frac{v_1^2}{c^2} \right)^{-1/2} = \left( \mathrm{m}(r) - \frac{\dot r_1^2 + r_1^2 \dot\phi^2}{c^2} \right)^{-1/2} \tag{57}$$

$$L = \gamma\, m\, r_1^2\, \dot\phi \tag{58}$$

and the relativistic Lagrangian reads

$$\mathscr{L} = -\frac{mc^2}{\gamma} + \frac{mMG}{r_1}. \tag{59}$$

Since the Lagrangian is simpler structured than for the observer coordinate system (Eq.(52)), the resulting Euler-Lagrange equations are simpler:

$$\boxed{\ddot\phi = \dot\phi\, \dot r \left( \frac{1}{\mathrm{m}(r_1)} \left( \frac{d}{dr_1}\, \mathrm{m}\,(r_1) + \frac{GM}{\gamma c^2\, {r_1}^2} \right) - \frac{2}{r} \right),} \tag{60}$$

$$\ddot{r} = \left(\frac{d}{dr_1}\,\mathrm{m}\,(r_1)\right)\left(c^2\,\left(\frac{1}{2} - \frac{1}{\gamma^2\;\mathrm{m}\,(r_1)}\right) - \frac{\dot{\phi}^2 \dot{r}_1^2}{\mathrm{m}\,(r_1)}\right) - \frac{GM\,\dot{\phi}^2}{\gamma\,c^2\;\mathrm{m}\,(r_1)} + \dot{\phi}^2\,r_1 - \frac{GM}{\gamma^3\,{r_1}^2\,\mathrm{m}\,(r_1)}. \tag{61}$$

The last term of $\ddot{\phi}$ and the two last terms of $\ddot{r}$ are the non-relativistic expressions, where the gravitational potential has a factor of $1/\gamma^3$ as already observerd in UFT 415. In addition, the m function appears in the gravitational potential.

## 3.2 Results of numerical calculations

$$L_N = m\,r^2\,\dot{\phi} \tag{62}$$

$$E_N = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) - \frac{mMG}{r} \tag{63}$$

../Fig1.png

Figure 1: Exponential function m($r$) for three values of $b$.

../Fig2.png

Figure 2: Schwarzschild-like function m($r$) for three values of $\alpha$.

../Fig3.png

Figure 3: Orbit of relativistic motion with exponential m function.

../Fig4.png

Figure 4: $\gamma$ factor of motion with exponential m function.

../Fig5.png

Figure 5: Angular momenta of motion with exponential m function.

../Fig6.png

Figure 6: Total energy of motion with exponential m function.

../Fig7.png

Figure 7: Collapsing orbit of exponential m function.

../Fig8.png

Figure 8: $\gamma$ factor and velocity $v$ of the collapsing orbit with exponential m function.

../Fig9.png

Figure 9: Angular momenta of the collapsing orbit with exponential m function.

../Fig10.png

Figure 10: Total energies of the collapsing orbit with exponential m function.

../Fig11.png

Figure 11: Stable orbit with Scharzschild-like m function, $\alpha = 0$.

../Fig12.png

Figure 12: Collapsing orbit with Scharzschild-like m function, $\alpha = 0.003$.

../Fig13.png

Figure 13: $\gamma$ factor and velocity $v$ of collapsing orbit with Scharzschild-like m function, $\alpha = 0.003$.