H8(4): The Fundamental Definition of the Work Integral in m Theory

In the standard model, the work integral is defined:

$$W_{12} = T_2 - T_1 = \int_1^2 \underline{F} \cdot d\underline{r} \quad -(1)$$

and is the difference in kinetic energies T_2 and T_1 in states 2 and 1. Here $\underline{F}$ is the force. If state 1 defines a particle at rest then:

$$W = T = \int \underline{F} \cdot d\underline{r} \quad -(2)$$

In the previous note it was shown that if:

$$\underline{F}_1 = \frac{d\underline{p}_1}{dt} = \frac{d}{dt}\left(\gamma m \dot{\underline{r}}_1\right) \quad -(3)$$

then in the standard model,

$$W = T = \int \frac{d}{dt}\left(\gamma m \dot{\underline{r}}_1\right) \cdot \dot{\underline{r}}_1 \, dt \quad -(4)$$

$$= m \int_0^{v_1} d(\gamma v_1)$$

Integrating by parts:

$$\int_0^{v_1} v_1 \, d(\gamma v_1) = \gamma v_1^2 - \int_0^{v_1} \gamma v_1 \, dv_1 \quad -(5)$$

where

$$\gamma = \left(m(r_1) - \frac{v_1^2}{c^2} \right)^{-1/2} \quad -(6)$$

It follows that:

$$\frac{d}{dv_1}\left(m(r_1) - \frac{v_1^2}{c^2} \right)^{1/2} = \gamma v_1 \quad -(7)$$

So:

$$m \int_0^{v_1} \gamma v_1 \, dv_1 = mc^2 \left(n(r_1) - \frac{v_1^2}{c^2} \right)^{1/2} \Bigg|_0^{v_1} \quad -(8)$$

This follows from:

$$\frac{d\gamma}{dv_1} = \frac{d}{dv_1} \left(n(r_1) - \frac{v_1^2}{c^2} \right)^{1/2}$$

$$= \frac{v_1}{c^2} \left(n(r_1) - \frac{v_1^2}{c^2} \right)^{-1/2} \quad -(9)$$

It has been assumed that:

$$\frac{dn(r_1)}{dv_1} = 0. \quad -(10)$$

The reason for this is that the general spherically symmetric spacetime is defined as:

$$ds^2 = g_{aa}(a,b)\,da^2 + g_{ab}(a,b)(da\,db + db\,da) + g_{bb}(a,b)\,db^2 + r^2(a,b)\,d\Omega^2 \quad -(11)$$

in spherical polar coordinates, where a and b are <u>coordinates</u>, and $r^2(a,b)$ is an undetermined function. The coordinate function $n(r_1)$ therefore does not depend on the velocity v_1 of a particle, and eq. (10) follows.

From eq. (8):

$$m \int_0^{v_1} \gamma v_1 \, dv_1 = mc^2 \left(n(r_1) - \frac{v_1^2}{c^2} \right)^{1/2} - n(r_1)^{1/2} mc^2$$

$-(12)$

) So the rest energy in n space is:

$$\boxed{E_0 = n(r_1)^{1/2} mc^2} \quad -(13)$$

This was first used in Eq. (32) of Note 417(7), Q.E.D.

Therefore the relativistic kinetic energy of n space is:

$$T = \gamma m v_1^2 + \frac{mc^2}{\gamma} - n(r_1)^{1/2} mc^2 \quad -(14)$$

Now use:

$$\gamma = \left(n(r_1) - \frac{v_1^2}{c^2} \right)^{-1/2} \quad -(15).$$

So

$$\frac{1}{\gamma^2} = n(r_1) - \frac{v_1^2}{c^2} \quad -(16)$$

and

$$v_1^2 = c^2 \left(n(r_1) - \frac{1}{\gamma^2} \right) \quad -(17)$$

So

$$T = \gamma mc^2 \left(n(r_1) - \frac{1}{\gamma^2} \right) + \frac{mc^2}{\gamma} - n(r_1)^{1/2} mc^2 \quad -(18)$$

i.e.

$$\boxed{T = n(r_1) \gamma mc^2 - n(r_1)^{1/2} mc^2} \quad -(19)$$

The total relativistic energy in n space is

$$E = n(r_1) \gamma mc^2 \quad -(20)$$

) The geodesic method of Note 416(3) shows that for free particle:

$$\frac{dE}{dt} = 0 \quad -(21)$$

and

$$\frac{dL}{dt} = 0 \quad -(22)$$

where

$$L = \gamma m r_1^2 \dot{\phi} \quad -(23)$$

, the angular momentum. The hamiltonian is:

$$H = \gamma m(r_1) mc^2 - \frac{mMG}{r_1} \quad -(24)$$

for a bound particle, and is also conserved:

$$\frac{dH}{dt} = 0 \quad -(25)$$

The equations of motion of m theory are always:

$$\frac{dH}{dt} = 0 \quad -(26)$$

and

$$\frac{dL}{dt} = 0. \quad -(27)$$

The m theory has been shown to be rigorously and completely self consistent. The force (3) is consistent with the hamiltonian and relativistic kinetic energy. The rest energy (13) is consistent with that used in UFT 417. The classical kinetic energy is recovered when:

$$T = m(r_1)\gamma mc^2 - m(r_1)^{1/2} mc^2 \xrightarrow[m(r_1) \to 1]{} (\gamma - 1) mc^2 \xrightarrow[v^2 \ll c^2]{} \frac{1}{2} mv^2 \quad -(28)$$