418(1): Lagrangian and Hamiltonian in the Classical Limit

Consider the relativistic lagrangian of the most general spherically symmetric spacetime in the coordinate system (r_1, ϕ):

$$\mathcal{L} = -mc^2 \left(m(r) - \frac{1}{c^2} \dot{\underline{r}}_1 \cdot \dot{\underline{r}}_1 \right)^{1/2} + \frac{mMG}{r_1} \quad -(1)$$

The Euler Lagrange equation is:

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}_1} = \frac{\partial \mathcal{L}}{\partial \underline{r}_1} = \underline{\nabla} \mathcal{L} = \frac{\partial \mathcal{L}}{\partial r_1} \underline{e}_r \quad -(2)$$

From these equations the relativistic force equation in m space is:

$$\underline{F}_1 = \frac{d\underline{p}_1}{dt} = -\left(\frac{mc^2}{2} \gamma \frac{dm(r)}{dr_1} - \frac{mMG}{r_1^2} \right) \underline{e}_r \quad -(3)$$

where

$$\underline{p}_1 = \gamma m \dot{\underline{r}}_1 \quad -(4)$$

is the relativistic momentum in m space. The Hooke / Newton inverse square law is:

$$\underline{F}_{HN} = -\frac{mMG}{r_1^2} \underline{e}_r \quad -(5)$$

and the vacuum force is

$$\underline{F}(\text{vac}) = -\frac{mc^2}{2} \gamma \frac{dm(r)}{dr_1} \underline{e}_r \quad -(6)$$

$$= \frac{\gamma mc^2 m(r)^{3/2}}{r \frac{dm(r)}{dr} - 2m(r)} \frac{dm(r)}{dr} \underline{e}_r$$

The Lorentz factor in m space is:

$$\gamma = \left(m(r) - \frac{1}{c^2} \dot{\underline{r}}_1 \cdot \dot{\underline{r}}_1 \right)^{-1/2} \quad -(7)$$

Now consider the classical limit of eq. (1):

$$\mathcal{L} = -mc^2\left(n(r_1)\left(1 - \frac{\dot{\underline{r}}_1 \cdot \dot{\underline{r}}_1}{n(r_1)c^2}\right)\right)^{1/2} + \frac{mMG}{r_1} \quad -(8)$$

$$= -mc^2 n(r_1)^{1/2}\left(1 - \frac{\dot{\underline{r}}_1 \cdot \dot{\underline{r}}_1}{n(r_1)c^2}\right)^{1/2} + \frac{mMG}{r_1}$$

$$\sim -mc^2 n(r_1)^{1/2}\left(1 - \frac{1}{2}\frac{\dot{\underline{r}}_1 \cdot \dot{\underline{r}}_1}{n(r_1)c^2}\right) + \frac{mMG}{r_1}$$

if $$\dot{\underline{r}}_1 \cdot \dot{\underline{r}}_1 \ll n(r_1)c^2 \quad -(9)$$

So the classical limit of the lagrangian is:

$$\mathcal{L} = \frac{1}{2} m \frac{\dot{\underline{r}}_1 \cdot \dot{\underline{r}}_1}{n(r_1)^{1/2}} + \frac{mMG}{r_1} - n(r_1)^{1/2} mc^2 \quad -(10)$$

$$= \frac{1}{2} m \frac{\dot{\underline{r}} \cdot \dot{\underline{r}}}{n(r)^{3/2}} + n(r)^{1/2}\frac{mMG}{r} - n(r)^{1/2} mc^2$$

In the same approximation (9) the hamiltonian is

$$H = \frac{1}{2} m \frac{\dot{\underline{r}} \cdot \dot{\underline{r}}}{n(r)^{3/2}} - n(r)^{1/2}\frac{mMG}{r} + n(r)^{1/2} mc^2 \quad -(11)$$

... is UFT447.

In the limit

$$n(r) \rightarrow 1 \quad -(12)$$

... hamiltonian (11) reduces to:

$$H = T + U + E_0 \quad -(13)$$

... here

$$E_0 = mc^2 \quad -(14)$$

3) Eq (13) is the well known definition of the hamiltonian in Minkowski spacetime (Marion and Thornton eq. (14.115) of the third edition. The definition includes the rest energy E_0. The usual classical hamiltonian is recovered from:

$$H_0 = T + U = H - E_0 \quad -(15)$$

The usual classical lagrangian is recovered using:

$$\mathcal{L}_0 = T - U = \mathcal{L} + E_0 \quad -(16)$$

It follows that:

$$\mathcal{L}_0 = \frac{1}{2} m \frac{\dot{\underline{r}} \cdot \dot{\underline{r}}}{m(r)^{3/2}} + m(r)^{1/2} \frac{mMG}{r} \quad -(17)$$

in the approximation (9).

The Euler Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}_0}{\partial \dot{\underline{r}}}\right) = \frac{\partial \mathcal{L}_0}{\partial \underline{r}} = \underline{\nabla} \mathcal{L}_0 \quad -(18)$$

$$-(19)$$

produces the force equation:

$$\boxed{\underline{F} = \frac{d}{dt}\left(\frac{m \dot{\underline{r}}}{m(r)^{3/2}}\right) = mMG \frac{\partial}{\partial r}\left(\frac{m(r)^{1/2}}{r}\right) \underline{e}_r}$$

So:

$$\underline{F} = m\left(\frac{\ddot{\underline{r}}}{m(r)^{3/2}} + \frac{d}{dt}\left(\frac{1}{m(r)^{3/2}}\right)\dot{\underline{r}}\right) \quad -(20)$$

$$= -\frac{mMG}{r^2}\left(m(r)^{1/2} - \frac{r}{2m(r)^{1/2}} \frac{dm(r)}{dr}\right)\underline{e}_r$$

In this equation the Newton second law in $m(r)$ space on the left hand side is balanced by

1) The inverse square law in n space:

$$\underline{F}(\text{inv. square}) = -m(r)^{1/2} \frac{n M G}{r^2} \underline{e}_r \quad -(21)$$

2) The vacuum force:

$$\underline{F}(\text{vac}) = \frac{n M G}{2 r m(r)^{1/2}} \frac{d m(r)}{dr} \underline{e}_r \quad -(22)$$

In the exact classical limit:

$$m(r) = 1 \quad -(23)$$

and eq. (20) reduces to the Newtonian:

$$\underline{F} = \frac{d\underline{p}}{dt} = -\frac{n M G}{r^2} \underline{e}_r \quad -(24)$$

In the limit (23):

$$\underline{F}(\text{vac}) = \underline{0} \quad -(25)$$

self consistently.

Newtonian dynamics and special relativity do not produce a vacuum force.

The fundamental equations governing the theory of the past few papers can be summarized by:

$$\frac{dH}{dt} = 0 \quad -(26)$$

and

$$\frac{dL}{dt} = 0 \quad -(27)$$

where the hamiltonian is

$$H = m(r)\gamma m c^2 - m(r)^{1/2} \frac{n M G}{r} \quad -(28)$$

and where the angular momentum is

$$\underline{L} = \frac{\gamma m c^2}{m(r)} \dot{\phi} \, \underline{k} \quad -(29)$$

$$\gamma = \left(m(r) - \frac{v^2}{m(r) c^2} \right)^{-1/2} \quad -(30)$$

In plane polar coordinates:

$$v^2 = \dot{\underline{r}} \cdot \dot{\underline{r}} = \dot{r}^2 + r^2 \dot{\phi}^2 \quad -(31)$$

In these equations:

$$m(r) = m(r(t)) \quad -(32)$$

so

$$\frac{d}{dt} m(r(t)) \neq 0 \quad -(33)$$

The m theory is the most general theory of orbits and dynamics. Its infinitesimal line element is:

$$ds^2 = c^2 d\tau^2 = m(r) c^2 dt^2 - \frac{dr^2}{m(r)} - r^2 d\phi^2 \quad -(34)$$

where m is any function of r. Eq. (34) defines the most general spherically symmetric spacetime.

It has been shown in recent papers that self consistency and rigorous conservation of energy and angular momentum require the use of the coordinate system (r_1, ϕ), where:

$$r_1 = \frac{r}{m(r)^{1/2}} \quad -(35)$$

The lagrangian in m space must be defined in (r_1, ϕ) as follows:

$$\mathcal{L} = -mc^2 \left(m(r_1) - \frac{1}{c^2} \dot{\underline{r}}_1 \cdot \dot{\underline{r}}_1 \right)^{1/2} + \frac{mMG}{r_1} \quad -(36)$$

The lagrangian (36) introduces startlingly new physics