417(6): The Relation between m Theory and Frame Rotation Theory

This relation is calculated straightforwardly by comparison of infinitesimal line elements. The infinitesimal line element of m theory is:

$$ds^2 = c^2 d\tau^2 = m(r) c^2 dt^2 - \frac{dr^2}{m(r)} - r^2 d\phi^2 \quad - (1)$$

and the infinitesimal line element of rotating frame theory is:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi'^2 \quad - (2)$$

where

$$d\phi' = d\phi + \omega dt \quad - (3)$$

Here ω is the angular velocity of frame rotation. As shown in Note 410(7), the retrograde precession of the S2 star system can be explained by:

$$d\phi' = d\phi - \omega dt \quad - (4)$$

The fundamental infinitesimal line element of spherical spacetime, eq. (1), can give rise to spacetime rotation by solving eqs. (1) and (2) simultaneously for $m(r)$. The advantage of this method is that orbital precession can be found in terms of $m(r)$.

In these equations the orbital linear velocity is

$$v_\phi = \omega r \quad - (5)$$

where

$$\omega = \frac{d\phi}{dt} \quad - (6)$$

From eq. (3):

$$d\phi'^2 = (d\phi + \omega dt)^2$$
$$= d\phi^2 + \omega^2 dt^2 + 2\omega d\phi dt \quad - (7)$$

and from eq. (4):

$$d\phi'^2 = (d\phi - \omega\, dt)^2 = d\phi^2 + \omega^2 dt^2 - 2\omega\, d\phi\, dt \quad -(8)$$

From eq. (7) in eq. (2):

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - 2\omega r^2 d\phi\, dt - \omega^2 r^2 dt^2$$
$$= (c^2 - v_\phi^2)\, dt^2 - 2\omega r^2 d\phi\, dt - dr^2 - r^2 d\phi^2 \quad -(9)$$

From Eq. (6):

$$d\phi = \omega\, dt \quad -(10)$$

so:

$$2\omega r^2 d\phi\, dt = 2\omega^2 r^2 dt^2 = 2 v_\phi^2 dt^2 \quad -(11)$$

Therefore eq. (2) becomes:

$$ds^2 = (c^2 - 3v_\phi^2)\, dt^2 - dr^2 - r^2 d\phi^2 \quad -(12)$$

From eq. (8) in eq. (2):

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 + 2\omega r^2 d\phi\, dt - \omega^2 r^2 dt^2$$
$$= (c^2 - v_\phi^2)\, dt^2 + 2\omega r^2 d\phi\, dt - dr^2 - r^2 d\phi^2$$
$$= (c^2 - v_\phi^2)\, dt^2 + 2 v_\phi^2 dt^2 - dr^2 - r^2 d\phi^2$$
$$= (c^2 + v_\phi^2)\, dt^2 - dr^2 - r^2 d\phi^2 \quad -(13)$$

Eq. (12) gives forward precession and eq. (13) gives retrograde precession. For forward precession in the classical limit:

$$r = \frac{\alpha}{1 + \epsilon \cos(\phi + \omega t)} \quad -(14)$$

and precession:

$$\Delta\phi = \omega T \quad -(15)$$

where T is the time taken for one orbit. For retrograde precession in the classical limit:

$$r = \frac{\alpha}{1 + \epsilon \cos(\phi - \omega t)} \quad -(16)$$

and:

$$\Delta\phi = -\omega T \quad -(17)$$

So the recent observation of retrograde precession in the S2 star is easily explained by frame rotation theory.

It can be shown by comparison of eqs. (1) and (12) eqns (1) and (13), that the frame rotation originates in m theory. Comparing eqs. (1) and (12):

$$m(r)c^2 dt^2 - \frac{dr^2}{m(r)} = \left(c^2 - 3v_\phi^2\right)dt^2 - dr^2 \quad -(18)$$

so

$$m(r) - \frac{1}{c^2}\frac{1}{m(r)}\left(\frac{dr}{dt}\right)^2 = 1 - \frac{3v_\phi^2}{c^2} - \frac{1}{c^2}\left(\frac{dr}{dt}\right)^2 \quad -(19)$$

and

$$\frac{3v_\phi^2}{c^2} = 1 - m(r) + \frac{1}{c^2}\left(\frac{1}{m(r)} - 1\right)\left(\frac{dr}{dt}\right)^2 \quad -(20)$$

If

$$\frac{dr}{dt} \ll c \quad -(21)$$

then:

$$\boxed{m(r) \sim 1 - \frac{3v_\phi^2}{c^2}} \quad -(22)$$

for forward precession.

Similarly, for retrograde precession:

$$\boxed{m(r) \sim 1 + \frac{v_\phi^2}{c^2}} \quad -(23)$$