417(3): Work Done by the Vacuum Force

Consider the vacuum force of Note 417(4):

$$\underline{F} = \left(\frac{\gamma m c^2 m(r)}{r \frac{dm(r)}{dr} - 2m(r)} \right)^{3/2} \frac{dm(r)}{dr} \underline{e}_r \quad -(1)$$

The work done by this force is:

$$W_{12} = \int_1^2 \underline{F} \cdot d\underline{r} = T_2 - T_1 = U_1 - U_2 \quad -(2)$$

where $T_2 - T_1$ is the change in kinetic energy and $U_1 - U_2$ is the change in potential energy. Eq. (2) is consistent with the fact that the hamiltonian H is a constant of motion:

$$H = T_1 + U_1 = T_2 + U_2 \quad -(3)$$

so

$$U_1 - U_2 = T_2 - T_1 \quad -(4)$$

The equation:

$$U_1 - U_2 = \int \underline{F} \cdot d\underline{r} \quad -(5)$$

is satisfied by:

$$\underline{F} = -\underline{\nabla} U \quad -(6)$$

so

$$U = -\int F \, dr \quad -(7)$$

if

$$U = U_1, \; U_2 = 0 \quad -(8)$$

The potential energy of the vacuum is therefore given by the integral:

$$U = -\int \frac{\gamma m c^2 m(r)^{3/2}}{r\frac{dm(r)}{dr} - 2m(r)} \frac{dm(r)}{dr}\, dr \quad -(9)$$

Considering:

$$T_2 - T_1 = U_1 - U_2 \quad -(10)$$

the potential energy U imparts the kinetic energy T to matter, for example the mass m of a planet. So

$$\boxed{T = \int \frac{\gamma m c^2 m(r)^{3/2}}{r\frac{dm(r)}{dr} - 2m(r)} \frac{dm(r)}{dr}\, dr} \quad -(10a)$$

and this is energy from the vacuum.

It would be interesting to integrate equation (10) numerically. It corresponds to the vacuum force:

$$\underline{F}(\text{vac}) = \frac{\gamma m c^2 m(r)^{3/2}}{r\frac{dm(r)}{dr} - 2m(r)} \frac{dm(r)}{dr} \quad -(11)$$

At the point:

$$r\frac{dm(r)}{dr} = 2m(r) \quad -(12)$$

the energy from the vacuum becomes infinite. It would be very interesting to graph this approach to infinity. The spin connection is defined by:

$$\underline{F}(\text{vac}) = -\frac{mMG}{r}\,\underline{\Omega} \quad -(13)$$

) so the spin connection can also become infinite.

To extend this approach to circuit theory, consider the equation:

$$\underline{F} = \frac{d\underline{p}}{dt} = -\frac{e^2}{4\pi\epsilon_0 r}\underline{e}_r \quad -(14)$$

where

$$\underline{p} = \frac{\gamma m \dot{\underline{r}}}{n(r)^{1/2}} \quad -(15)$$

and

$$\gamma = \left(n(r) - \frac{v^2}{n(r)c^2} \right)^{-1/2} \quad -(16)$$

then the vacuum force on one electron is given by eq. (11), in which m would be the mass of the electron. The energy imparted to the electron from the vacuum would be eq. (10).

In plane polar coordinates the velocity v^2 is

$$v^2 = \dot{r}^2 + r^2\dot{\phi}^2 \quad -(17)$$

in which $\dot{r}$ and $\dot{\phi}$ must be found from solving:

$$\frac{dH}{dt} = 0 \quad -(18)$$

$$\frac{dL}{dt} = 0. \quad -(19)$$

The hamiltonian is:

$$H = \gamma n(r) mc^2 - \frac{e^2}{4\pi\epsilon_0 r} \quad -(20)$$

) and the angular momentum is

$$L = \frac{\gamma m r^2 \dot{\phi}}{n(r)} \quad -(21)$$

Eqs. (18) and (19) are solved numerically, so the force and energy or the elements of a circuit can be found. These are the vacuum force and energy. They can be transferred to a suitably defined circuit. The force and energy from the vacuum are maximized when

$$\boxed{r \frac{dn(r)}{dr} = 2n(r)} \quad -(22)$$

This is the condition for maximum energy from spacetime. The function $n(r)$ is defined by:

$$ds^2 = c^2 d\tau^2 = n(r) c^2 dt^2 - \frac{dr^2}{n(r)} - r^2 d\phi^2 \quad -(23)$$

in plane polar coordinates (r, ϕ). In general, $n(r)$ can be any function, because in theory is not constrained by the incorrect Einstein field equation. So the energy transferred to the circuit can be maximized by attempting to use condition (22), by "vacuum engineering".

Finally an idea of the effect of non Minkowski spacetime can be obtained from the fact that if:

$$n(r) = 1 \quad -(24)$$

) there is no vacuum energy or force in Minkowski spacetime.
If the Minkowski lagrangian is considered:

$$\mathcal{L} = -mc^2\left(1 - \frac{v^2}{c^2}\right)^{1/2} - U \quad -(25)$$

$$\sim -mc^2\left(1 - \frac{1}{2}\frac{v^2}{c^2} + \cdots\right) - U$$

$$= \frac{1}{2}mv^2 - mc^2 - U$$

$$= \mathcal{L}_c - mc^2$$

here

$$\mathcal{L}_c = \frac{1}{2}mv^2 - U \quad -(26)$$

is the classical lagrangian.

In the m space:

$$\mathcal{L} = -mc^2\left(m(r) - \frac{v^2}{m(r)c^2}\right)^{1/2} - U$$

$$= -mc^2\left(m(r)\left(1 - \frac{v^2}{m^2(r)c^2}\right)\right)^{1/2} - U$$

$$\sim -mc^2 m(r)^{1/2}\left(1 - \frac{1}{2}\frac{v^2}{m^2(r)c^2} + \cdots\right) - U$$

$$= \frac{1}{2}m\frac{v^2}{m(r)^{3/2}} - m(r)^{1/2}mc^2 - U \quad -(27)$$

So the rest energy is modified to

$$E_0 = m(r)^{1/2}mc^2 \quad -(28)$$

and the classical kinetic energy is modified to

$$T = \frac{1}{2} \frac{m v^2}{n(r)^{3/2}} \qquad -(25)$$

A classical theory of orbits could be based therefore on the lagrangian:

$$H = \frac{1}{2} \frac{m v^2}{n(r)^{3/2}} + U \qquad -(26)$$

More generally, any classical theory of dynamics can be transformed into m theory by using the lagrangian (26). For example, a classical computer simulation can be based on the lagrangian (26), and animated. This gives classical computer simulation in the general, spherically symmetric spacetime. Another example is Lamb shift theory where the original vacuum force is:

$$\underline{F}(vac) = m \frac{d^2 \delta \underline{r}}{dt^2} = -e\underline{E} \qquad -(27)$$

By comparison of Eqs. (1) and (27) it can be seen that

$$\frac{d^2 \delta r}{dt^2} = \frac{\gamma c^2 n(r)^{3/2}}{r \frac{dn(r)}{dr} - 2n(r)} \frac{dn(r)}{dr} = -eE \qquad -(28)$$

so $n(r)$ is responsible for the Lamb shift.