Final Version of Note 417(1).

From eq. (18) onwards:

$$\frac{dm(r)}{dr_1} = \frac{dm(r)}{dr}\frac{dr}{dr_1} \quad -(1)$$

where:

$$r_1 = \frac{r}{m(r)^{1/2}} \quad -(2)$$

Therefore

$$\frac{dr_1}{dr} = \frac{d}{dr}\left(\frac{r}{m(r)^{1/2}}\right) \quad -(3)$$

$$= \left(m^{1/2}(r) - \left(\frac{d}{dr}m(r)^{1/2}\right)r\right) \Big/ m(r)$$

Let

$$y = m(r)^{1/2} \quad -(4)$$

$$:= f^{1/2}$$

then

$$\frac{dy}{dr} = \frac{df}{dr}\frac{dy}{df} = \frac{1}{2}f^{-1/2}\frac{dm(r)}{dr} \quad -(5)$$

So

$$\frac{d}{dr}m(r)^{1/2} = \frac{1}{2}m(r)^{-1/2}\frac{dm(r)}{dr} \quad -(6)$$

It follows that:

$$\frac{dr_1}{dr} = \frac{1}{m(r)}\left(m^{1/2}(r) - \frac{1}{2}r\,m(r)^{-1/2}\frac{dm(r)}{dr}\right) \quad -(7)$$

so

$$\frac{dr}{dr_1} = m(r)\left(m^{1/2}(r) - \frac{1}{2}r\,m(r)^{-1/2}\frac{dm(r)}{dr}\right)^{-1} \quad -(8)$$

and

$$\boxed{F(vac) = -\frac{mc^2}{2}\,\gamma\, m(r)\frac{dm(r)}{dr}\left(m^{1/2}(r) - \frac{1}{2}r\,m(r)^{-1/2}\frac{dm(r)}{dr}\right)^{-1}} \quad -(9)$$

i.e.

$$\boxed{F(\text{vac}) = \frac{-\frac{mc^2}{2}\,\gamma\, m^{1/2}(r)\,\frac{dm(r)}{dr}}{1-\frac{1}{2}\,r\,\frac{dm(r)}{dr}}} \quad -(10)$$

This is maximized when:

$$\frac{1}{2}\,r\,\frac{dm(r)}{dr} = 1 \quad -(11)$$

At this point the vacuum force approaches infinity.

If for example:

$$m(r) = 1 - \frac{r_0}{r} + \frac{\alpha}{r^2} \quad -(12)$$

then:

$$\frac{dm(r)}{dr} = \frac{r_0}{r^2} - \frac{2\alpha}{r^3} \quad -(13)$$

and the vacuum force is maximized when:

$$\frac{1}{2}\,r\left(\frac{r_0}{r^2} - \frac{2\alpha}{r^3}\right) = 1 \quad -(14)$$

i.e.

$$r^2 - \frac{r r_0}{2} + \alpha = 0 \quad -(15)$$

so:

$$r = \frac{1}{4}\left(r_0 - \left(r_0^2 - 4\alpha\right)^{1/2}\right) \quad -(16)$$

The vacuum force is maximized at this point.

) Note that when

$$\alpha = 0 \quad - (18)$$

and

$$n = 1 - \frac{r_0}{r} \quad - (19)$$

the vacuum force is maximized at the origin:

$$r = 0 \quad - (20)$$

Therefore this analysis shows that in the general spherical spacetime, there exists an infinite vacuum force under well defined conditions.