7(2): Computation of the Force due to Spherical Spacetime

From Note 417(1) the magnitude of the force is:

$$F(\text{vac}) = -\frac{mc^2}{2}\frac{\gamma}{m^2(r)}\frac{dm(r)}{dr}\left(m(r) - \frac{dm(r)}{dr}\right) \quad -(1)$$

where

$$\gamma = \left(m(r) - \frac{\dot{r}^2 + r^2\dot{\phi}^2}{m(r)c^2}\right)^{-1/2} \quad -(2)$$

The quantities $\dot{r}$ and $\dot{\phi}$ are computed from the equations of motion:

$$\frac{dH}{dt} = 0 \quad -(3)$$

and

$$\frac{dL}{dt} = 0 \quad -(4)$$

Computer algebra applied to eqs. (3) and (4) as in Section 3 of UFT416 gives:

$$\ddot{r} = \frac{dm(r)}{dr}\left(c^2 m(r) + \frac{MG}{2\gamma^3 r\, m(r)^{1/2}} - \frac{3c^2}{2\gamma^2}\right) - \frac{1}{m(r)}\frac{dm(r)}{dr}\dot{\phi}^2 r^2\left(2 - \frac{MG}{2\gamma c^2 m(r)^{1/2}}\right) - \frac{MG\dot{\phi}^2}{\gamma c^2 m(r)^{1/2}} + \dot{\phi}^2 r - \frac{MG\, m(r)^{1/2}}{\gamma^3 r^2} \quad -(5)$$

and

$$\ddot{\phi} = \dot{\phi}\dot{r}\left(\frac{1}{m(r)}\frac{dm(r)}{dr}\left(2 - \frac{MG}{2\gamma c^2 r\, m(r)^{1/2}}\right) + \frac{MG}{\gamma c^2 r^2 m(r)^{1/2}} - \frac{2}{r}\right) \quad -(6)$$

Eqs. (5) and (6) are solved simultaneously to give $\dot{r}$ and $\dot{\phi}$, and so γ can be computed.

2) In Minkowski spacetime:

$$m(r) = 1 \quad -(7)$$

so eqns. (5) and (6) reduce to:

$$\ddot{r} = \frac{MG\dot{\phi}^2}{\gamma c^2} + \dot{\phi}^2 r - \frac{MG}{\gamma^3 r^2} \quad -(8)$$

and

$$\ddot{\phi} = \dot{\phi}\dot{r}\left(\frac{MG}{\gamma c^2 r^2} - \frac{2}{r}\right) \quad -(9)$$

From previous UFT papers it is known that eqs. (8) and (9) give orbital precession. Eqs. (5) and (6) give both precession and shrinking, and replace "Big Bang" and "black holes", replacing them with new cosmologies.

In the classical limit:

$$\gamma \rightarrow 1, \; c \rightarrow \infty \quad -(10)$$

Eqs. (8) and (9) reduce to:

$$\ddot{r} = \dot{\phi}^2 r - \frac{MG}{r^2} \quad -(11)$$

and

$$\ddot{\phi} = -2\frac{\dot{\phi}\dot{r}}{r} \quad -(12)$$

Eq. (11) is the Leibniz force equation:

$$\ddot{r} - \dot{\phi}^2 r = -\frac{MG}{r^2} \quad -(13)$$

and eq. (12) is:

$$\ddot{\phi} r + 2\dot{\phi}\dot{r} = 0 \quad -(14)$$

which is the conservation equation of the classical angular momentum:

$$L = mr^2\dot{\phi}, \quad -(15)$$

$$\frac{dL}{dt} = 0 \quad -(16)$$

Eqs. (13) and (14) are those of Newtonian dynamics and orbit theory, Q.E.D. They give non precessing conic section orbits.

Note that the acceleration due to gravity $\underline{g}$ is

$$\underline{g} = (\ddot{r} - r\dot{\phi}^2)\underline{e}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\underline{e}_\phi$$

in the plane polar coordinate system (r, ϕ).$^{(17)}$ So eqs. (13) and (14) are the result of:

$$\underline{F} = m\underline{g} = -\frac{mMG}{r^2}\underline{e}_r \quad -(18)$$

which is the Newtonian equation of orbits, Q.E.D.

Eqs. (8) and (9) can be written as:

$$\ddot{r} - \dot{\phi}^2 r = -\left(\frac{MG}{\gamma^3 r^2} - \frac{MG\dot{\phi}^2}{\gamma c^2}\right) \quad -(19)$$

$$= -\frac{MG}{\gamma}\left(\frac{1}{\gamma^2 r^2} - \frac{\dot{\phi}^2}{c^2}\right)$$

and

$$\ddot{\phi} + \frac{2\dot{\phi}\dot{r}}{r} = MG\left(\frac{\dot{\phi}\dot{r}}{\gamma c^2 r^2}\right) \quad -(20)$$

i.e.

$$\ddot{\phi} r + 2\dot{\phi}\dot{r} = \left(\frac{\dot{\phi}\dot{r}}{\gamma c^2 r}\right) MG \quad -(21)$$

The relativistic equivalents of eqs. (13) and (14) are therefore eqs. (19) and (21), which produce precession but no shrinking, and no vacuum force, and are not sufficient to cover Big Bang and black hole theory. Eqs. (19) and (21) do not produce a cosmology, they produce orbits only.

4) Eqs. (5) and (6) can be written as:

$$\ddot{r} - \dot{\phi}^2 r = \frac{dm(r)}{dr}\left(c^2 m(r) + \frac{MG}{2\gamma^3 r\, m(r)^{1/2}} - \frac{3c^2}{2\gamma^2}\right) - \frac{1}{m(r)}\frac{dm(r)}{dr}\dot{\phi}^2 r^2\left(2 - \frac{MG}{2\gamma c^2 m(r)^{1/2}}\right) - MG\left(\frac{m(r)^{1/2}}{\gamma^3 r^2} + \frac{\dot{\phi}^2}{\gamma c^2 m(r)^{1/2}}\right) \quad -(22)$$

and

$$r\ddot{\phi} + 2\dot{\phi}\dot{r} = r\dot{\phi}\dot{r}\left(\frac{1}{m(r)}\frac{dm(r)}{dr}\left(2 - \frac{MG}{2\gamma c^2 r\, m(r)^{1/2}}\right) + \frac{MG}{\gamma c^2 r^2 m(r)^{1/2}}\right) \quad -(23)$$

Eqs. (22) and (23) generalize the Newtonian equations (13) and (14) to give new cosmologies, and any observable orbit. They give precession, retrograde precession, shrinking and a new event horizon theory, and give the vacuum force (1).

The Newtonian infinitesimal line element is

$$ds^2 = v^2 dt^2 = dr^2 + r^2 d\phi^2 \quad -(24)$$

The infinitesimal line element of Minkowski spacetime is

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - (dr^2 + r^2 d\phi^2) \quad -(25)$$

the infinitesimal line element of the most general spherically symmetric spacetime is:

$$ds^2 = c^2 d\tau^2 = m(r) c^2 dt^2 - \frac{dr^2}{m(r)} - r^2 d\phi^2 \quad -(26)$$

Eq. (24) gives rise to eqs. (13) and (14). Eq. (25)

5) gives rise to eqs. (19) and (21), and eq. (26) gives rise to eqs. (19) and (21).

Eqs. (13) and (14) are equivalent to:

$$\frac{dH}{dt} = 0 \quad -(27)$$

and

$$\frac{dL}{dt} = 0 \quad -(28)$$

where

$$H = \frac{1}{2} m \dot{r}^2 - \frac{mMG}{r} \quad -(29)$$

and

$$L = m r^2 \dot{\phi} \quad -(30)$$

are the classical hamiltonian and angular momentum. To see this denote:

$$v = \dot{r} \quad -(31)$$

Eq. (29) is written in Cartesian coordinates, so:

$$\underline{v} \cdot \underline{v} = \underline{\dot{r}} \cdot \underline{\dot{r}} = \dot{r}^2 = v^2 \quad -(32)$$

It follows that:

$$H = \frac{1}{2} m v^2 - \frac{mMG}{r} \quad -(33)$$

and

$$\frac{dH}{dt} = \frac{1}{2} m \frac{dv^2}{dt} - mMG \frac{d}{dt}\left(\frac{1}{r}\right) = 0 \quad -(34)$$

Use:

$$\frac{dv^2}{dt} = \frac{dv^2}{dv} \frac{dv}{dt} = 2 v \dot{v} \quad -(35)$$

and

$$\frac{d}{dt}\left(\frac{1}{r}\right) = \frac{d}{dr}\left(\frac{1}{r}\right) \frac{dr}{dt} = -\frac{1}{r^2} v \quad -(36)$$

so eq. (34) gives

$$\ddot{r} = \dot{v} = -\frac{MG}{r^2} \quad -(37)$$

in Cartesian coordinates, Q.E.D.

In Minkowski spacetime:

$$H = \gamma m c^2 - \frac{mMG}{r} \quad -(38)$$

and

$$\frac{dH}{dt} = \frac{d}{dt}\left(\gamma_1 mc^2 - \frac{mMG}{r}\right) = 0 \quad -(39)$$

Also

$$L = \gamma_1 mr^2\dot{\phi} \quad -(40)$$

and

$$\frac{dL}{dt} = \frac{d}{dt}\left(\gamma_1 mr^2\dot{\phi}\right) = 0 \quad -(41)$$

The equations of motion are eqs. (39) and (41).

In the most general spherically symmetric spacetime:

$$H = m(r)\,\gamma_2 mc^2 - \frac{mMG}{r} \quad -(42)$$

and

$$L = \frac{\gamma_2 mr^2\dot{\phi}}{m(r)} \quad -(43)$$

The equations of motion from eqs. (42) and (43) are again:

$$\frac{dH}{dt} = 0, \quad \frac{dL}{dt} = 0 \quad -(44)$$

Here:

$$\gamma_1 = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(45)$$

and

$$\gamma_2 = \left(m(r) - \frac{v^2}{m(r)c^2}\right)^{-1/2} \quad -(46)$$

Also

$$v^2 = \dot{r}^2 + r^2\dot{\phi}^2 \quad -(47)$$