416(2): The Self Consistent Coordinate System

Consider the plane polar coordinate system (r_1, ϕ), where:

$$r_1 = \frac{r}{n(r)^{1/2}} \quad -(1)$$

The lagrangian is then:

$$\mathcal{L} = -mc^2\left(n(r) - \frac{1}{c^2}\left(\dot{r}_1^2 + r_1^2\dot{\phi}^2\right)\right)^{1/2} + \frac{mMG}{r_1} \quad -(2)$$

where

$$\dot{r}_1 = \frac{\dot{r}}{n(r)^{1/2}} \quad -(3)$$

The lagrangian has the fundamental property:

$$\frac{\partial \mathcal{L}}{\partial t} = 0 \quad -(4)$$

(Marion and Thornton). The linear momentum is:

$$\underline{p}_1 = \frac{\partial \mathcal{L}}{\partial \underline{\dot{r}}_1} = \gamma m \underline{\dot{r}}_1 = \frac{\gamma m \underline{\dot{r}}}{n(r)^{1/2}} \quad -(5)$$

This result is also obtained from kinematics with:

$$\underline{r} = \frac{r}{n(r)^{1/2}}\underline{e}_r. \quad -(6)$$

as derived in UFT415 from fundamental geometry.

The angular momentum from eq. (2) is:

$$L = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \gamma m r_1^2 \dot{\phi} \quad -(7)$$

$$= \frac{\gamma m r^2 \dot{\phi}}{n(r)}$$

From eqs. (5) and (6):

$$\underline{L} = \underline{r} \times \underline{p} = \frac{\gamma m r^2 \dot{\phi}}{n(r)}\underline{k} \quad -(8)$$

so the theory is self consistent, Q.E.D.

) From Lagrangian dynamics, the Leibniz equation in

- space is:

$$\underline{\dot{p}}_1 = \frac{\partial \mathcal{L}}{\partial \underline{r}_1} \quad - (9)$$

i.e

$$\frac{d}{dt}\left(\frac{\gamma m \dot{\underline{r}}}{n(r)^{1/2}}\right) = -\frac{mMG}{r_1^2} \quad - (10)$$

i.e.

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}_1}\right) = \frac{\partial \mathcal{L}}{\partial \underline{r}_1} \quad - (11)$$

which is the Euler Lagrange equation in m space.
Note that the Lagrangian (2) can be written as:

$$\mathcal{L} = -mc^2\left(n(r) - \frac{1}{c^2 n(r)}\left(\dot{r}^2 + r^2\dot{\phi}^2\right)\right)^{1/2} + \frac{n^{1/2}(r)mMG}{r} \quad - (12)$$

so the potential energy is changed to:

$$U(r) = -\frac{mMG}{r_1} = -n^{1/2}(r)\frac{mMG}{r} \quad - (13)$$

In a curved spacetime such as m spacetime the distance between m and M is $r / n^{1/2}(r)$. For small curvature:

$$r_1 \rightarrow r \quad - (14)$$

From the Euler Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\right) = \frac{\partial \mathcal{L}}{\partial \phi} \quad - (15)$$

it follows that L is conserved:

$$\frac{dL}{dt} = 0 \quad - (16)$$

- in UFT415.

*) The only difference between the hamiltonian and lagrangian used in UFT415 is that the plane polar coordinate system is changed to (r_1, ϕ). In the coordinate system there is no need for an $n^{1/2}(r)$ factor in eq. (57) of UFT415.

In Hamiltonian dynamics the canonical equations are

$$\underline{\dot{r}} = \frac{\partial H}{\partial \underline{p}} \quad -(17)$$

and

$$\underline{\dot{p}} = -\frac{\partial H}{\partial \underline{r}} \quad -(18)$$

where H is the hamiltonian. In the (r_1, ϕ) coordinate system these are changed to:

$$\underline{\dot{r}}_1 = \frac{\partial H}{\partial \underline{p}_1} \quad -(19)$$

and

$$\underline{\dot{p}}_1 = -\frac{\partial H}{\partial \underline{r}_1} \quad -(20)$$

In Hamiltonian dynamics:

$$H = H(q, p, t) \quad -(21)$$

In Lagrangian dynamics:

$$\mathcal{L} = \mathcal{L}(q, \dot{q}, t) \quad -(22)$$

(Marion and Thornton).

Using

$$H = n(r)\,\gamma mc^2 - \frac{mMG}{r_1} \quad -(23)$$

$$= n(r)\left(n(r) - \frac{v_1^2}{c^2}\right)^{-1/2} mc^2 - \frac{mMG}{r_1}$$

) it follows that

$$\dot{r}_1 = \frac{\partial H}{\partial p_1} = \frac{p_1}{m^2 c^2} \gamma m(r) m c^2 = m(r) \gamma \frac{p_1}{m} \quad -(24)$$

where

$$p_1 = \frac{v}{m(r)^{1/2}} \quad -(25)$$

so

$$v = \frac{\dot{r}}{m(r)^{1/2}} \quad -(26)$$

Q.E.D.

Eq. (26) is the kinematic result used in UFT415,

o the constant of motion:

$$H = m(r) \gamma m c^2 - \frac{m M G}{r_1} \quad -(27)$$

is self consistently defined, Q.E.D.

The hamiltonian H has the property:

$$\frac{dH}{dt} = 0 \quad -(28)$$

Similarly the Lagrangian (2) has the property:

$$\frac{\partial \mathcal{L}}{\partial t} = 0 \quad -(29)$$
