416(1): Introduction of the Spin Connection to the New Orbit Theory

In UFT415 it was shown that the equations of motion:

$$\frac{dH}{dt} = 0 \quad -(1)$$

and

$$\frac{dL}{dt} = 0 \quad -(2)$$

can describe essentially every orbit in the universe. In this note we begin to discuss the role of the spin connection and the vacuum force.

In eq. (1):

$$H = m(r)\gamma mc^2 - \frac{mMG}{r} \quad -(3)$$

and

$$L = \gamma m r^2 \dot{\phi} \quad -(4)$$

in which:

$$m(r) := m(r(t)) \quad -(5)$$

The $\gamma$ factor is a development of the Lorentz factor $\gamma$ to $m$ space, the most general spherically symmetric space:

$$\gamma = \left( m(r) - \frac{v^2}{m(r)c^2} \right)^{-1/2} \quad -(6)$$

In plane polar coordinates:

$$v^2 = \dot{r}^2 + r^2 \dot{\phi}^2 \quad -(7)$$

In these equations $H$ is the conserved hamiltonian, $L$ is the conserved angular momentum, and $m(r)$ is defined by the infinitesimal line element:

$$ds^2 = c^2 d\tau^2 = m(r)c^2 dt^2 - \frac{dr^2}{m(r)} - r^2 d\phi^2 \quad -(8)$$

) Note carefully that in space, the position vector is:

$$\underline{r} = \frac{r}{m(r)^{1/2}}\,\underline{e}_r \qquad -(9)$$

If the metric is a stationary metric then $m(r)$ is stationary and

$$\underline{v} = \dot{\underline{r}} = \frac{1}{m(r)^{1/2}}\left(\dot{r}\,\underline{e}_r + r\dot{\phi}\,\underline{e}_\phi\right) \qquad -(10)$$

and we obtain eq. (6).

Similarly, the Minkowski metric is $\mathrm{diag}(1, -1, -1, -1)$ and is a spherically symmetric stationary metric. Clearly, $\mathrm{diag}(1, -1, -1, -1)$ does not change with time, and the Minkowski metric produces the infinitesimal line element:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 \qquad -(11)$$

in the limit

$$m(r) \rightarrow 1 \qquad -(12)$$

Stationary metrics have time like Killing vectors (Carroll online notes for "Spacetime and Geometry: An Introduction to General Relativity", chapter 7.

In general, in a spherically symmetric spacetime:

$$ds^2 = g_{aa}(a,b)\,da^2 + g_{ab}(a,b)\,(da\,db + db\,da) + g_{bb}(a,b)\,db^2 + r^2(a,b)\,d\Omega^2 \qquad -(13)$$

in spherical polar coordinates. Here $r(a,b)$ is an undetermined function and $(a,b)$ are coordinates. This can be reduced to:

$$ds^2 = c^2 d\tau^2 = \exp(2\alpha)\,dt^2 - \exp(2\beta)\,dr^2 - r^2 d\phi^2 \qquad -(14)$$

for plane polar coordinates. Eq. (14) is true for a stationary metric if $\alpha$ and $\beta$ are time independent. Eq. (14) is true independently of the Einstein field equation. Therefore

3) in the theory we make the choice:

$$\exp(2\alpha) = m(r) \quad -(15)$$

$$\exp(2\beta) = \frac{1}{m(r)} \quad -(16)$$

in which r is related to a coordinate. Therefore:

$$ds^2 = c^2 d\tau^2 = m(r)\, c^2 dt^2 - d\underline{r} \cdot d\underline{r} \quad -(17)$$

where

$$d\underline{r} \cdot d\underline{r} = \frac{dr^2}{m(r)} + r^2 d\phi^2 \quad -(18)$$

In general:

$$d\underline{r} = \frac{\partial \underline{r}}{\partial r} dr + \frac{\partial \underline{r}}{\partial \phi} d\phi \quad -(19)$$

in plane polar coordinates, so:

$$d\underline{r} \cdot d\underline{r} = \left( \frac{\partial \underline{r}}{\partial r} dr + \frac{\partial \underline{r}}{\partial \phi} d\phi \right) \cdot \left( \frac{\partial \underline{r}}{\partial r} dr + \frac{\partial \underline{r}}{\partial \phi} d\phi \right) \quad -(20)$$

$$= \frac{dr^2}{m(r)} + r^2 d\phi^2$$

As in UFT 415:

$$\frac{\partial \underline{r}}{\partial r} = \frac{1}{m(r)^{1/2}} \underline{e}_r \quad -(21)$$

$$\frac{\partial \underline{r}}{\partial \phi} = r\, \underline{e}_\phi . \quad -(22)$$

So

$$\boxed{\underline{r} = \frac{r}{m(r)^{1/2}} \underline{e}_r} \quad -(23)$$

For a stationary, spherically symmetric, metric:

$$\underline{r} = r \frac{\underline{e}_r}{m(r)^{1/2}} \quad -(24)$$

) Here $n(r)$ regarded as a coordinate function, is time independent

The unit vector $\underline{e}_r$ has been replaced by $\underline{e}_r / n(r)$.

It follows that:

$$\underline{v} = \underline{\dot{r}} = \dot{r}\frac{\underline{e}_r}{n(r)^{1/2}} + \frac{d}{dt}\left(\frac{\underline{e}_r}{n(r)^{1/2}}\right) r \quad -(25)$$

$$= \frac{1}{n(r)^{1/2}}\left(\dot{r}\,\underline{e}_r + r\dot{\phi}\,\underline{e}_\phi\right)$$

Therefore:

$$\boxed{\gamma = \left(n(r) - \frac{\dot{r}^2 + r^2\dot{\phi}^2}{n(r)c^2}\right)^{-1/2}} \quad -(26).$$

This result is consistent with eqns. ~~(1) and (2)~~ and is true for any $n(r)$.

In deriving equations of motion, $r$ must be regarded as a function of time:

$$r = r(t) \quad -(27)$$

so

$$\gamma(t) = \left(n(r(t)) - \frac{\dot{r}^2 + r^2\dot{\phi}^2}{n(r(t))c^2}\right)^{-1/2} \quad -(28)$$

Eqs. (1), (2) and (28) lead to self-consistent conservation of the hamiltonian and angular momentum. The hamiltonian is defined as:

$$H_1 := m(r(t))\,\gamma(t)\,mc^2 - mc^2 - \frac{mMG}{r(t)} \quad -(29)$$

nd the angular momentum as:

$$L = \gamma(t)\, m\, r(t)\, \dot{\phi}(t) \quad -(30)$$

Computer algebra by Dr. Horst Eckardt shows that:

$$\ddot{\phi}(t) = \dot{\phi}\dot{r}\left(\frac{2}{m(r)}\frac{dm(r)}{dt} + \frac{MG}{\gamma c^2 r^2 m(r)} - \frac{2}{r}\right) \quad -(30)$$

and

$$\ddot{r}(t) = \frac{1}{m(r)}\frac{dm(r)}{dr}\left(\left(\frac{1}{2\gamma^2 m(r)} + 1\right)\dot{r}^2 + \dot{\phi}^2 r^2\left(\frac{1}{2\gamma^2 m(r)} - 1\right)\right)$$

$$+ \frac{c^2}{\gamma^2}\left(\frac{1}{2\gamma^2 m(r)} - 1\right)\frac{dm(r)}{dr} - \frac{MG\dot{r}^2}{\gamma c^2 m(r)}$$

$$+ \dot{\phi}^2 r - \frac{MG}{\gamma^3 r^2} \quad -(31)$$

in which:

$$m(r) = m(r(t)) \quad -(32)$$

These equations are more general than the Einstein field equation and are rigorously correct. By Birkhoff's theorem the Einstein equation in a stationary spherically symmetric spacetime can only produce the Schwarzschild metric with:

$$m(r) = 1 - \frac{2MG}{c^2 r} \quad -(33)$$

The force equation corresponding to eq. (1)

and (2) is E(E2 covariant:

$$\underline{F} = -\underline{\nabla}\,\underline{\Phi} + \underline{\Omega}_r\,\underline{\Phi} \quad -(34)$$

$$= -\underline{\nabla}\,\underline{\Phi} + \underline{F}(\mathrm{vac})$$

where

$$\underline{F}(\mathrm{vac}) = \underline{\Omega}_r\,\underline{\Phi} \quad -(35)$$

is the vacuum force.

The force will be derived from the hamiltonian in the next note

---