

GENERAL ORBIT THEORY IN SPHERICALLY SYMMETRIC SPACETIME.

by

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ABSTRACT

The most general type of orbit theory is developed in spherically symmetric spacetime. This theory is developed from the fact that the relativistic hamiltonian (H) and relativistic angular momentum (L) are constants of motion. The coordinate system is defined for the general orbit and the equations of motion solved numerically. The resulting orbits precess and in general can decrease, so that a particle m orbiting a particle M eventually collides with M . This theory can describe all observable orbits without the use of the Einstein field equation.

Keywords: ECE2 theory, general orbits in spherically symmetric spacetime.

4FT416



1. INTRODUCTION

In immediately preceding papers of this series {1 - 41} an ECE2 covariant theory of orbits has been developed with the aim of describing precession and orbit shrinking without the incorrect Einstein field equation. The latter fails experimentally in S star systems by an order of magnitude and has been refuted in nearly a hundred different ways in the ECE and ECE2 theories. In Section 2 a rigorously self consistent coordinate system is defined and used to define the lagrangian and hamiltonian. This is the only coordinate system that is rigorously self consistent. The hamiltonian and angular momentum in this coordinate system are rigorously self consistent constants of motion, and this property is used to construct a powerful and simple new cosmology in which orbits can in general both precess and shrink, as observed for example in binary pulsars. The new cosmology can describe S star systems, in which Einsteinian general relativity (EGR) fails by an order of magnitude. S star systems entirely refute the claimed precision of EGR and indeed refute the entire twentieth century thought in gravitational physics. In Section 3 an extensive numerical and graphical analysis is given of the first results from this new cosmology.

This paper is a short synopsis of extensive calculations in the notes accompanying UFT416 on www.aiaa.us and www.upitec.org. Note 416(1) gives a short review of the properties of the most general spherically symmetric line elements and metrics, and gives the equations of motion of UFT415 using the plane polar coordinate system (r, ϕ) . Note 416(2) develops the rigorously self consistent coordinate system used to produce the orbits of Section 3. This coordinate system is the one defined by the most general spherically symmetric spacetime and must always be used. Note 416(3) double checks Note 416(2) using the geodesic method.

2. RIGOROUSLY SELF CONSISTENT THEORY

Consider the plane polar coordinate system (r_1, ϕ) where

$$r_1 = \frac{r}{m(r)^{1/2}} \quad - (1)$$

In this system of coordinates the infinitesimal line element of spherically symmetric spacetime is:

$$ds^2 = c^2 d\tau^2 = m(r) c^2 dt^2 - dr_1^2 - r_1^2 d\phi^2 \quad - (2)$$

in which the Newtonian velocity v is defined by:

$$v^2 dt^2 = dr_1^2 + r_1^2 d\phi^2 \quad - (3)$$

It follows that the Lorentz factor is generalized to:

$$\gamma = \left(m(r) - \frac{v^2}{c^2} \right)^{-1/2} \quad - (4)$$

The free particle kinetic lagrangian {1 - 41} is:

$$\mathcal{L} = \frac{1}{2} mc^2 = \frac{1}{2} m \left(\frac{ds}{d\tau} \right)^2 = \frac{1}{2} m g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} \quad - (5)$$

where $g_{\mu\nu}$ is the metric and:

$$x^\mu = (ct, r_1) \quad - (6)$$

is the position four vector. It follows from Eqs. (2) and (6) that: - (7)

$$\mathcal{L} = \frac{1}{2} m \left(m(r) c^2 \left(\frac{dt}{d\tau} \right)^2 - \left(\frac{dr_1}{d\tau} \right)^2 - r_1^2 \left(\frac{d\phi}{d\tau} \right)^2 \right) = \frac{1}{2} m g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}$$

Therefore:

$$\frac{1}{2} m g_{00} \frac{dx^0}{d\tau} \frac{dx^0}{d\tau} = \frac{1}{2} m \dot{m}(r) c^2 \left(\frac{d\tau}{d\tau} \right)^2 \quad - (8)$$

$$\frac{1}{2} m g_{11} \frac{dx^1}{d\tau} \frac{dx^1}{d\tau} = \frac{1}{2} m \left(\frac{dr_1}{d\tau} \right)^2 \quad - (9)$$

$$\frac{1}{2} m g_{22} \frac{dx^2}{d\tau} \frac{dx^2}{d\tau} = \frac{1}{2} m \left(\frac{d\phi}{d\tau} \right)^2 r_1^2 \quad - (10)$$

The Hamilton principle of least action is:

$$\delta \int \mathcal{L} d\tau = 0 \quad - (11)$$

and the Euler Lagrange equation is:

$$\frac{d}{d\tau} \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu} = \frac{\partial \mathcal{L}}{\partial x^\mu} \quad - (12)$$

where:

$$\dot{x}^\mu = \frac{dx^\mu}{d\tau} \quad - (13)$$

From the Euler Lagrange equation:

$$\frac{d}{d\tau} \frac{\partial \mathcal{L}}{\partial \dot{x}^0} = \frac{d}{d\tau} \frac{\partial \mathcal{L}}{\partial \left(\frac{dt}{d\tau} \right)} = 0 \quad - (14)$$

it follows that:

$$\frac{dE}{d\tau} = 0 \quad - (15)$$

where the total relativistic kinetic energy E of the free particle is

$$E = \frac{\partial \mathcal{L}}{\partial \left(\frac{dt}{d\tau} \right)} = m(r) mc^2 \frac{dt}{d\tau} = m(r) \gamma mc^2 \quad - (16)$$

and from Eq. (15) is a constant of motion of a free particle. The hamiltonian of an interacting particle of mass m is therefore:

$$H = E + U. \quad - (17)$$

Note carefully that in the curved m space the potential energy of interaction between m and M in an orbit is:

$$U = - \frac{mM G}{r_1} \quad - (18)$$

Here M is the mass of the particle about which m orbits, and G is Newton's constant. The hamiltonian is a constant of motion in general:

$$\frac{dH}{dt} = 0 \quad - (19)$$

The Euler Lagrange equation:

$$\frac{d}{d\tau} \frac{\partial \mathcal{L}}{\partial \dot{x}^i} = \frac{d}{d\tau} \frac{\partial \mathcal{L}}{\partial \left(\frac{dr_1}{d\tau} \right)} = 0 \quad - (20)$$

gives:

$$\frac{dp_i}{d\tau} = 0 \quad - (21)$$

where

$$p_i = \frac{\partial \mathcal{L}}{\partial \left(\frac{dr_1}{d\tau} \right)} = m \frac{dr_1}{d\tau} \quad - (22)$$

p_i is the conserved linear momentum of a free particle in the most general spherically

symmetric spacetime. By definition:

$$p_1 = m \frac{dr_1}{d\tau} = \frac{\gamma}{m(r)^{1/2}} m \frac{dr}{dt} \quad - (23)$$

as in UFT415, so the coordinate system and theory is rigorously self consistent, Q. E. D.

Finally the Euler Lagrange equation:

$$\frac{d}{d\tau} \frac{\partial \mathcal{L}}{\partial \dot{x}^2} = \frac{d}{d\tau} \frac{\partial \mathcal{L}}{\partial \left(\frac{d\phi}{d\tau} \right)} = 0 \quad - (24)$$

gives:

$$\frac{dL}{d\tau} = 0 \quad - (25)$$

where:

$$L = m r_1^2 \frac{d\phi}{d\tau} = \frac{\gamma m r^2}{m(r)} \frac{d\phi}{dt} \quad - (26)$$

is the conserved angular momentum in the most general spherically symmetric spacetime.

This is the same angular momentum as in UFT415, but the plane polar coordinate system

(r, ϕ) does not give the correct linear momentum derived from fundamental kinematic

considerations as in UFT415. The equations of motion of the new cosmology are therefore:

$$\frac{dH}{d\tau} = 0 \quad - (27)$$

and

$$\frac{dL}{d\tau} = 0 \quad - (28)$$

Finally use:

$$\frac{dH}{d\tau} = \frac{dH}{dt} \frac{dt}{d\tau} = \gamma \frac{dH}{dt} \quad - (29)$$

and

$$\frac{dL}{d\tau} = \frac{dL}{dt} \frac{dt}{d\tau} = \gamma \frac{dL}{dt} \quad - (30)$$

to find that:

$$\frac{dH}{dt} = 0 \quad - (31)$$

and

$$\frac{dL}{dt} = 0 \quad - (32)$$

These are integrated numerically in Section 3 to give any observable orbit. The numerical method checks that H and L are rigorously conserved, so the numerical and analytical techniques are correct and H and L are rigorously conserved, Q. E. D.

The orbital lagrangian in (r_1, ϕ) is

$$\begin{aligned} \mathcal{L} &= -mc^2 \left(m(r) - \frac{1}{c^2} \dot{\underline{r}}_1 \cdot \dot{\underline{r}}_1 \right)^{1/2} + \frac{\gamma m \Gamma}{r_1} \\ &= -mc^2 \left(m(r) - \frac{1}{c^2} (\dot{r}_1^2 + r_1^2 \dot{\phi}^2) \right)^{1/2} + \frac{\gamma m \Gamma}{r_1} \end{aligned} \quad - (33)$$

and has the well known fundamental property:

$$\frac{d\mathcal{L}}{dt} = 0 \quad - (34)$$

The linear momentum from Eq. (33) is:

$$\underline{p}_1 = \frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}_1} = \gamma m \dot{\underline{r}}_1 = \frac{\gamma m}{m(r)^{1/2}} \dot{\underline{r}} \quad - (35)$$

and is the same as the result obtained in UFT415 from the fundamental definition of the position vector \underline{r} in the most general spherically symmetric space:

$$\underline{r}_1 = \underline{r} \frac{\underline{e}_r}{r} = \frac{r}{m(r)^{1/2}} \underline{e}_r \quad - (36)$$

The conserved angular momentum from Eq. (33) is:

$$L = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \gamma m r^2 \dot{\phi} = \frac{\gamma m r^2}{m(r)} \dot{\phi} \quad - (37)$$

which is the same as the result from the geodesic method, Eq. (26), Q. E. D.

Furthermore, from Eqs. (35) and (36):

$$\underline{L} = \underline{r}_1 \times \underline{p}_1 = \frac{\gamma m r^2}{m(r)} \frac{d\phi}{dt} \underline{k} \quad - (38)$$

which is the same result again for the conserved angular momentum, giving a triple cross check on the angular momentum.

The Leibniz equation in the most general spherically symmetric space is:

$$\underline{\dot{p}}_1 = \frac{\partial \mathcal{L}}{\partial \underline{r}_1} \quad - (39)$$

i. e. :

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}_1} \right) = \frac{\partial \mathcal{L}}{\partial \underline{r}_1} \quad - (40)$$

and

$$\frac{d}{dt} \left(\frac{\gamma m \dot{\underline{r}}}{m(r)^{1/2}} \right) = -m \frac{M G}{r_1^3} \underline{r}_1 \quad - (41)$$

The equations of motion (31) and (32) are developed numerically in

Section 3 and in the coordinate system (r, ϕ) can be written as:

$$\ddot{r} = \left(\frac{dm(r)}{dr} \right) \left(c^2 m(r) + \frac{M G}{2 \gamma^3 r m(r)^{1/2}} - \frac{3c^2}{2 \gamma^2} \right) - \frac{dm(r)}{dr} \frac{\dot{\phi}^2 r^2}{m(r)} \left(2 - \frac{m G}{2 \gamma c^2 r m(r)^{1/2}} \right) - \frac{M G \dot{\phi}^2}{\gamma c^2 m(r)^{1/2}} + \dot{\phi}^2 r - \frac{m G m(r)^{1/2}}{2 \gamma^2} \quad - (42)$$

— (43)

and

$$\ddot{\phi} = \frac{\dot{\phi} \dot{r}}{m(r)} \frac{dm(r)}{dr} \left(2 - \frac{M G}{2 \gamma c^2 r m(r)^{1/2}} \right) + \frac{M G}{\gamma c^2 r^2 m(r)^{1/2}} - \frac{2}{r}$$

These are integrated as simultaneous equations giving a vast amount of new information about any observable orbit. A small sample of such information is presented in Section 3.

3. SOME RESULTS FROM EQS. (42) AND (43).

Section by Dr. Horst Eckardt.

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension and the staff of AIAS and others for many interesting discussions. Dave Burleigh, CEO of Annexa Inc., is thanked for voluntary posting, site maintenance and feedback maintenance. Alex Hill is thanked for many translations, and Robert Cheshire and Michael Jackson for broadcasting and video preparation.

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