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(%i1) kill(all);
(%o0) done
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1 Quartic equation (62)

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(%i1) E: 2*a*x^4-r*x^3=v^2*r*a/(M*G);
(E) 2 a x^4 - r x^3 = \frac{a r v^2}{G M}
(%i2) E1: solve(E, x)$
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(%i3) first(E1);
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(%o3) x = -sqrt(-
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$$\frac{\sqrt{3} r^3}{8 a^2} \sqrt{\frac{48 G M a^2 \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{2/3} + 3 G M r^2 \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3} - 32 a^2 r v^2}{G M \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3}}}$$

$$\left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3} + \frac{2 r v^2}{3 G M \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3} + \frac{r^2}{8 a^2}} \Big/ 2 -$$

$$\sqrt{\frac{48 G M a^2 \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{2/3} + 3 G M r^2 \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3} - 32 a^2 r v^2}{G M \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3}}}$$

$$8 \sqrt{3} a} + \frac{r}{8 a}$$

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(%i4) second(E1);
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(%o4) x = sqrt(-
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$$\frac{\sqrt{3} r^3}{8 a^2} \sqrt{\frac{48 G M a^2 \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{2/3} + 3 G M r^2 \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3} - 32 a^2 r v^2}{G M \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3}}}$$

$$\left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3} + \frac{2 r v^2}{3 G M \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3} + \frac{r^2}{8 a^2}} \Big/ 2 -$$

$$\sqrt{\frac{48 G M a^2 \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{2/3} + 3 G M r^2 \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3} - 32 a^2 r v^2}{G M \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3}}}$$

$$8 \sqrt{3} a} + \frac{r}{8 a}$$

(%i5) third(E1);

(%o5) $x = -\text{sqrt}(\sqrt{3} r^3$

$$\frac{8 a^2 \sqrt{\frac{48 G M a^2 \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{2/3} + 3 G M r^2 \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3} - 32 a^2 r v^2}{\left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3} + \frac{2 r v^2}{3 G M \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3} + \frac{r^2}{8 a^2}}}{2} + \frac{G M \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3}}{8 \sqrt{3} a} + \frac{r}{8 a}}$$

(%i6) fourth(E1);

(%o6) $x = \text{sqrt}(\sqrt{3} r^3$

$$\frac{8 a^2 \sqrt{\frac{48 G M a^2 \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{2/3} + 3 G M r^2 \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3} - 32 a^2 r v^2}{\left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3} + \frac{2 r v^2}{3 G M \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3} + \frac{r^2}{8 a^2}}}{2} + \frac{G M \left(\frac{r v^2 \sqrt{\frac{r (2048 a^4 v^2 + 27 G M r^3)}{G M}}}{16 3^{3/2} G M a^2} - \frac{r^3 v^2}{16 G M a^2} \right)^{1/3}}{8 \sqrt{3} a} + \frac{r}{8 a}}$$