1 Coordinates

```
(%i1)  kill(all);
(%o0)  done
```

```
(%i1)  depends([phi,r,omega,T,U,L,gamma],t,Phi,r);
(%o1)  [φ(t), r(t), ω(t), T(t), U(t), L(t), Γ(t), Φ(r)]
```

```
(%i2)  assume(c>0);
(%o2)  [c>0]
```

First version

1 Kinetic energy

```
(%i3)  T: -m*c^2/gamma;
       /*T: m*c^2*(gamma-1);*/
```

$$(T) \quad -\frac{c^2\, m}{\Gamma}$$

```
(%i4)  v2: diff(r,t)^2+r^2*diff(phi,t)^2;
```

$$(v2) \quad \left(\frac{\mathrm{d}}{\mathrm{d}\,t}\,r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\,\varphi\right)^2 r^2$$

```
(%i5)  gamma: 1/sqrt(1-v2/c^2);
```

$$(\text{gamma}) \quad \frac{1}{\sqrt{1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\,r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\,\varphi\right)^2 r^2}{c^2}}}$$

2 Potential energy

```
(%i6)  Phi: -M*G/r;
```

$$(\text{Phi}) \quad -\frac{G\,M}{r}$$

```
(%i7)  Ustd: m*Phi;
```

$$(\text{Ustd}) \quad -\frac{G\,M\,m}{r}$$

3 Lagrangian

```
(%i8)  La: ev(T)-Ustd;
```

$$(La) \quad \frac{G\,M\,m}{r}-c^2\, m\sqrt{1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\,r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\,\varphi\right)^2 r^2}{c^2}}$$

```
(%i9)  Etot: T+Ustd;
```

$$(\text{Etot}) \quad -\frac{c^2\, m}{\Gamma}-\frac{G\,M\,m}{r}$$

4 Lagrange equations

4.1 Definitions

```
(%i10)  D1(La,q) := diff(La, q)/*+Q[q]*/;
```

$$(\%o10) \quad \mathrm{D1}(La,q):=\frac{\mathrm{d}}{\mathrm{d}\,q}\,La$$

```
(%i11)  D2a(La,q) := diff(La, diff(q,t));/*only for constants of motion*/;
```

$$(\%o11) \quad \mathrm{D2a}(La,q):=\frac{\mathrm{d}}{\mathrm{d}\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\,q\right)}\,La$$

```
(%i12)  D2(La,q) := diff(diff(La, diff(q,t)),t);
```

$$(\%o12) \quad \mathrm{D2}(La,q):=\frac{\mathrm{d}}{\mathrm{d}\,t}\left(\frac{\mathrm{d}}{\mathrm{d}\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\,q\right)}\,La\right)$$

```
(%i13)  LE(La,q) := D1(La,q) = D2(La,q);
```

(%o13) $\mathrm{LE}(La,q):=\mathrm{D1}(La,q)=\mathrm{D2}(La,q)$

4.2 Evaluation

```
(%i14)  E11: LE(La,phi);
```

(E11) $$0=\frac{m\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)r^2\left(2\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}\,t^2}r\right)+2\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)+2\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}\,t^2}\varphi\right)r^2\right)}{2\,c^2\left(1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}\right)^{3/2}}+\frac{2\,m\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)r\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)}{\sqrt{1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}}}+\frac{m\left(\frac{\mathrm{d}^2}{\mathrm{d}\,t^2}\varphi\right)r^2}{\sqrt{1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}}}$$

```
(%i15)  E21: LE(La,r);
```

(E21) $$\frac{m\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r}{\sqrt{1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}}}-\frac{G\,M\,m}{r^2}=\frac{m\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)\left(2\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}\,t^2}r\right)+2\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)+2\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}\,t^2}\varphi\right)r^2\right)}{2\,c^2\left(1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}\right)^{3/2}}+\frac{m\left(\frac{\mathrm{d}^2}{\mathrm{d}\,t^2}r\right)}{\sqrt{1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}}}$$

4.3 Constants of motion

```
(%i16)  D2a(La, phi);
```

(%o16) $$\frac{m\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)r^2}{\sqrt{1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}}}$$

4.4 Resolving for highest derivatives of variables and final equations

```
(%i17)  GC: trigsimp(first(solve([E11,E21],
            [diff(phi,t,2), diff(r,t,2)])));
```

(GC) $$\left[\frac{\mathrm{d}^2}{\mathrm{d}\,t^2}\varphi=\frac{G\,M\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)\sqrt{-\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2-\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2+c^2}-2\,c^3\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)r\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)}{c^3\,r^2},\ \frac{\mathrm{d}^2}{\mathrm{d}\,t^2}r=\frac{\sqrt{-\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2-\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2+c^2}\left(G\,M\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2-G\,M\,c^2\right)+c^3\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^3}{c^3\,r^2}\right]$$

```
(%i18)  ratsubst(1/(c*%gamma), sqrt(-('diff(r,t,1))^2-('diff(phi,t,1))^2*r^2+c^2),GC);
```

(%o18) $$\left[\frac{\mathrm{d}^2}{\mathrm{d}\,t^2}\varphi=-\frac{\left(2\,\gamma\,c^4\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)r-G\,M\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)\right)\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)}{\gamma\,c^4\,r^2},\ \frac{\mathrm{d}^2}{\mathrm{d}\,t^2}r=\frac{G\,M\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\gamma\,c^4\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^3-G\,M\,c^2}{\gamma\,c^4\,r^2}\right]$$

```
(%i19)  expand(%);
```

(%o19) $$\left[\frac{\mathrm{d}^2}{\mathrm{d}\,t^2}\varphi=\frac{G\,M\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)}{\gamma\,c^4\,r^2}-\frac{2\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)}{r},\ \frac{\mathrm{d}^2}{\mathrm{d}\,t^2}r=\frac{G\,M\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2}{\gamma\,c^4\,r^2}+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r-\frac{G\,M}{\gamma\,c^2\,r^2}\right]$$

Second version

1 Kinetic energy

(%i20)
```
/*T: -m*c^2/gamma;*/
T: m*c^2*(gamma-1);
```

(T)
$$c^2\,m\left(\frac{1}{\sqrt{1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}}}-1\right)$$

(%i21)
```
v2: diff(r,t)^2+r^2*diff(phi,t)^2;
```

(v2)
$$\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2$$

(%i22)
```
gamma: 1/sqrt(1-v2/c^2);
```

(gamma)
$$\frac{1}{\sqrt{1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}}}$$

2 Lagrangian

(%i23)
```
La: ev(T)-Ustd;
```

(La)
$$c^2\,m\left(\frac{1}{\sqrt{1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}}}-1\right)+\frac{G\,M\,m}{r}$$

(%i24)
```
Etot: T+Ustd;
```

(Etot)
$$c^2\,m\left(\frac{1}{\sqrt{1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}}}-1\right)-\frac{G\,M\,m}{r}$$

2.1 Evaluation

(%i25)
```
E11: LE(La,phi);
```

(E11)
$$0=\frac{3\,m\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)r^2\left(2\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}\,t^2}r\right)+2\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)+2\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}\,t^2}\varphi\right)r^2\right)}{2\,c^2\left(1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}\right)^{5/2}}+\frac{2\,m\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)r\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)}{\left(1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}\right)^{3/2}}+\frac{m\left(\frac{\mathrm{d}^2}{\mathrm{d}\,t^2}\varphi\right)r^2}{\left(1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}\right)^{3/2}}$$

(%i26)
```
E21: LE(La,r);
```

(E21)
$$\frac{m\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r}{\left(1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}\right)^{3/2}}-\frac{G\,M\,m}{r^2}=\frac{3\,m\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)\left(2\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}\,t^2}r\right)+2\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)+2\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}\,t^2}\varphi\right)r^2\right)}{2\,c^2\left(1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}\right)^{5/2}}+\frac{m\left(\frac{\mathrm{d}^2}{\mathrm{d}\,t^2}r\right)}{\left(1-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}\,t}r\right)^2+\left(\frac{\mathrm{d}}{\mathrm{d}\,t}\varphi\right)^2 r^2}{c^2}\right)^{3/2}}$$

2.2 Constants of motion

```
(%i27)  D2a(La, phi);
```

$$(\%o27)\quad \frac{m\left(\frac{d}{dt}\varphi\right)r^2}{\left(1-\frac{\left(\frac{d}{dt}r\right)^2+\left(\frac{d}{dt}\varphi\right)^2 r^2}{c^2}\right)^{3/2}}$$

2.3 Resolving for highest derivatives of variables and final equations

```
(%i28)  GC: trigsimp(first(solve([E11/gamma^2,E21/gamm^2],
            [diff(phi,t,2), diff(r,t,2)])));
```

$$(GC)\quad [\frac{d^2}{dt^2}\varphi=-(\sqrt{-\left(\frac{d}{dt}r\right)^2-\left(\frac{d}{dt}\varphi\right)^2 r^2+c^2}\left(3\,G\,M\left(\frac{d}{dt}\varphi\right)\left(\frac{d}{dt}r\right)^3+\left(3\,G\,M\left(\frac{d}{dt}\varphi\right)^3 r^2-3\,G\,M\,c^2\left(\frac{d}{dt}\varphi\right)\right)\left(\frac{d}{dt}r\right)\right)+4\,c^3$$

$$\left(\frac{d}{dt}\varphi\right)r\left(\frac{d}{dt}r\right)^3+\left(4\,c^3\left(\frac{d}{dt}\varphi\right)^3 r^3+2\,c^5\left(\frac{d}{dt}\varphi\right)r\right)\left(\frac{d}{dt}r\right))/(2\,c^3\,r^2\left(\frac{d}{dt}r\right)^2+2\,c^3\left(\frac{d}{dt}\varphi\right)^2 r^4+c^5\,r^2),\frac{d^2}{dt^2}r=-($$

$$\sqrt{-\left(\frac{d}{dt}r\right)^2-\left(\frac{d}{dt}\varphi\right)^2 r^2+c^2}\left(G\,M\left(\frac{d}{dt}r\right)^4+\left(-G\,M\left(\frac{d}{dt}\varphi\right)^2 r^2-2\,G\,M\,c^2\right)\left(\frac{d}{dt}r\right)^2-2\,G\,M\left(\frac{d}{dt}\varphi\right)^4 r^4+G\,M\,c^2\left(\frac{d}{dt}\varphi\right)^2 r^2+G\,M\,c^4\right)$$

$$-2\,c^3\left(\frac{d}{dt}\varphi\right)^2 r^3\left(\frac{d}{dt}r\right)^2-2\,c^3\left(\frac{d}{dt}\varphi\right)^4 r^5-c^5\left(\frac{d}{dt}\varphi\right)^2 r^3)/(2\,c^3\,r^2\left(\frac{d}{dt}r\right)^2+2\,c^3\left(\frac{d}{dt}\varphi\right)^2 r^4+c^5\,r^2)]$$

```
(%i29)  ratsubst(1/(c*%gamma), sqrt(-('diff(r,t,1))^2-('diff(phi,t,1))^2*r^2+c^2),GC);
```

$$(\%o29)\quad [\frac{d^2}{dt^2}\varphi=-$$

$$\frac{\left(4\,\gamma\,c^4\left(\frac{d}{dt}\varphi\right)r+3\,G\,M\left(\frac{d}{dt}\varphi\right)\right)\left(\frac{d}{dt}r\right)^3+\left(4\,\gamma\,c^4\left(\frac{d}{dt}\varphi\right)^3 r^3+3\,G\,M\left(\frac{d}{dt}\varphi\right)^3 r^2+2\,\gamma\,c^6\left(\frac{d}{dt}\varphi\right)r-3\,G\,M\,c^2\left(\frac{d}{dt}\varphi\right)\right)\left(\frac{d}{dt}r\right)}{2\,\gamma\,c^4\,r^2\left(\frac{d}{dt}r\right)^2+2\,\gamma\,c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma\,c^6\,r^2},\frac{d^2}{dt^2}r=-($$

$$G\,M\left(\frac{d}{dt}r\right)^4+\left(-2\,\gamma\,c^4\left(\frac{d}{dt}\varphi\right)^2 r^3-G\,M\left(\frac{d}{dt}\varphi\right)^2 r^2-2\,G\,M\,c^2\right)\left(\frac{d}{dt}r\right)^2-2\,\gamma\,c^4\left(\frac{d}{dt}\varphi\right)^4 r^5-2\,G\,M\left(\frac{d}{dt}\varphi\right)^4 r^4-\gamma\,c^6\left(\frac{d}{dt}\varphi\right)^2 r^3+G$$

$$M\,c^2\left(\frac{d}{dt}\varphi\right)^2 r^2+G\,M\,c^4)/(2\,\gamma\,c^4\,r^2\left(\frac{d}{dt}r\right)^2+2\,\gamma\,c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma\,c^6\,r^2)]$$

(%i30) `expand(%);`

(%o30)

$$\left[\frac{d^2}{dt^2}\varphi=-\frac{4\gamma c^4\left(\frac{d}{dt}\varphi\right)r\left(\frac{d}{dt}r\right)^3}{2\gamma c^4 r^2\left(\frac{d}{dt}r\right)^2+2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma c^6 r^2}-\frac{3GM\left(\frac{d}{dt}\varphi\right)\left(\frac{d}{dt}r\right)^3}{2\gamma c^4 r^2\left(\frac{d}{dt}r\right)^2+2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma c^6 r^2}-\right.$$

$$\frac{4\gamma c^4\left(\frac{d}{dt}\varphi\right)^3 r^3\left(\frac{d}{dt}r\right)}{2\gamma c^4 r^2\left(\frac{d}{dt}r\right)^2+2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma c^6 r^2}-\frac{3GM\left(\frac{d}{dt}\varphi\right)^3 r^2\left(\frac{d}{dt}r\right)}{2\gamma c^4 r^2\left(\frac{d}{dt}r\right)^2+2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma c^6 r^2}-$$

$$\frac{2\gamma c^6\left(\frac{d}{dt}\varphi\right)r\left(\frac{d}{dt}r\right)}{2\gamma c^4 r^2\left(\frac{d}{dt}r\right)^2+2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma c^6 r^2}+\frac{3GMc^2\left(\frac{d}{dt}\varphi\right)\left(\frac{d}{dt}r\right)}{2\gamma c^4 r^2\left(\frac{d}{dt}r\right)^2+2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma c^6 r^2},\frac{d^2}{dt^2}r=-$$

$$\frac{GM\left(\frac{d}{dt}r\right)^4}{2\gamma c^4 r^2\left(\frac{d}{dt}r\right)^2+2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma c^6 r^2}+\frac{2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^3\left(\frac{d}{dt}r\right)^2}{2\gamma c^4 r^2\left(\frac{d}{dt}r\right)^2+2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma c^6 r^2}+$$

$$\frac{GM\left(\frac{d}{dt}\varphi\right)^2 r^2\left(\frac{d}{dt}r\right)^2}{2\gamma c^4 r^2\left(\frac{d}{dt}r\right)^2+2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma c^6 r^2}+\frac{2GMc^2\left(\frac{d}{dt}r\right)^2}{2\gamma c^4 r^2\left(\frac{d}{dt}r\right)^2+2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma c^6 r^2}+$$

$$\frac{2\gamma c^4\left(\frac{d}{dt}\varphi\right)^4 r^5}{2\gamma c^4 r^2\left(\frac{d}{dt}r\right)^2+2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma c^6 r^2}+\frac{2GM\left(\frac{d}{dt}\varphi\right)^4 r^4}{2\gamma c^4 r^2\left(\frac{d}{dt}r\right)^2+2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma c^6 r^2}+$$

$$\frac{\gamma c^6\left(\frac{d}{dt}\varphi\right)^2 r^3}{2\gamma c^4 r^2\left(\frac{d}{dt}r\right)^2+2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma c^6 r^2}-\frac{GMc^2\left(\frac{d}{dt}\varphi\right)^2 r^2}{2\gamma c^4 r^2\left(\frac{d}{dt}r\right)^2+2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma c^6 r^2}-$$

$$\left.\frac{GMc^4}{2\gamma c^4 r^2\left(\frac{d}{dt}r\right)^2+2\gamma c^4\left(\frac{d}{dt}\varphi\right)^2 r^4+\gamma c^6 r^2}\right]$$