44(9): Equations of Motion from the Minkowski Metric and ECE2 Covariant Metric.

These two metrics can be written as:

$$ds^2 = c^2 d\tau^2 = (c^2 - v_N^2)\, dt^2 \quad -(1)$$

Therefore as in UFT106 and UFT192:

$$mc^2 = mc^2 \left(\frac{dt}{d\tau}\right)^2 - m v_N^2 \left(\frac{dt}{d\tau}\right)^2 \quad -(2)$$

Here v_N is the Newtonian velocity, and the Lorentz factor is

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2}, \quad -(3)$$

and τ is the proper time. Therefore:

$$m^2 c^4 = \gamma^2 m^2 c^4 - \gamma^2 m^2 c^2 v_N^2 \quad -(4)$$

This is the Einstein energy equation where:

$$E = \gamma m c^2, \quad p = \gamma m \underline{v}, \quad E_0 = mc^2 \quad -(5)$$

Eq. (4) is valid for a free particle if the total energy and the hamiltonian are the same:

$$H = E = \gamma m c^2 \quad -(6)$$

and for a particle in orbit if:

$$H = E + U = \gamma m c^2 - \frac{mMG}{r} \quad -(7)$$

where

$$U = -\frac{mMG}{r} \quad -(8)$$

is the gravitational potential. Under all circumstances H is a constant of motion:

$$\frac{dH}{dt} = 0 \quad -(9)$$

The Minkowski metric is defined in a space with no torsion and no curvature, but the ECE2 metric is defined in general in a space with finite torsion and curvature. As in UFT199, the most general infinitesimal line element in a spherically symmetric spacetime is:

$$ds^2 = c^2 d\tau^2 = m(r)\, c^2 dt^2 - \frac{dr^2}{m(r)} - r^2 d\phi^2 \quad -(10)$$

and this is the basis for m theory. For the ECE2 theory, the infinitesimal line element can be extended to eq. (10). In the usual ECE2 theory:

$$m(r) = 1. \quad -(11)$$

and the infinitesimal line element (10) reduces to:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 \quad -(12)$$

This equation can be written as:

$$mc^2 = \frac{E^2}{mc^2} - m\left(\frac{dr}{d\tau}\right)^2 - mr^2\left(\frac{d\phi}{d\tau}\right)^2 - m\left(\frac{dr}{d\tau}\right)^2 \quad -(13)$$

Using the constant of motion:

$$L = mr^2 \frac{d\phi}{d\tau} = \gamma m r^2 \frac{d\phi}{dt} \quad -(14)$$

where L is the angular momentum, then:

$$m\left(\frac{dr}{d\tau}\right)^2 = \frac{E^2}{mc^2} - \frac{L^2}{mr^2} - mc^2 \quad -(15)$$

where:

$$\frac{dr}{d\tau} = \frac{dr}{d\phi}\frac{d\phi}{d\tau} = \frac{L}{mr^2}\frac{dr}{d\phi} \quad -(16)$$

It follows that:

$$\left(\frac{dr}{d\phi}\right)^2 = r^4\left(\frac{1}{b^2} - \left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right) \quad -(17)$$

where

$$a = \frac{L}{mc}, \quad b = \frac{Lc}{E} \quad -(18)$$

Eq. (17) follows also from the free particle hamiltonian:

$$E = H = \gamma mc^2 \quad -(19)$$

This follows from:

$$\gamma = \frac{E}{mc^2} = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} \quad -(20)$$

so

$$\frac{v_N^2}{c^2} = 1 - \frac{m^2c^4}{E^2} \quad -(21)$$

The Newtonian velocity is:

$$v_N^2 = \dot{r}^2 + r^2\dot{\phi}^2 \quad -(22)$$

Now use:

$$\frac{dr}{dt} = \frac{dr}{d\phi}\frac{d\phi}{dt} = \dot{\phi}\frac{dr}{d\phi} \quad -(23)$$

so

$$v_N^2 = \left(r^2 + \left(\frac{dr}{d\phi}\right)^2\right)\dot{\phi}^2 \quad -(24)$$

In Newtonian dynamics the constant angular momentum

$$L_0 = mr^2\dot{\phi} \quad -(25)$$

so

$$v_N^2 = \frac{L_0^2}{m^2}\left(\frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\phi}\right)^2\right) \quad -(26)$$

4) From Eqs. (20) and (26):

$$\frac{L_0^2}{m^2c^2r^4}\left(r^2+\left(\frac{dr}{d\phi}\right)^2\right) = 1 - \frac{m^2c^4}{E^2} \quad -(27)$$

So

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{m^2c^2r^4}{L_0^2}\left(1-\frac{m^2c^4}{E^2}\right) - r^2$$

$$= r^4\left(\frac{1}{b^2} - \frac{m^2c^4}{E^2}\frac{m^2c^2}{L_0^2}\right) - r^2 \quad -(28)$$

In this equation:

$$\frac{m^2c^4}{E^2} = \frac{1}{\gamma^2} \quad -(29)$$

and

$$L_0^2 = \frac{L^2}{\gamma^2} \quad -(30)$$

so

$$\left(\frac{dr}{d\phi}\right)^2 = r^4\left(\frac{1}{b^2}-\frac{1}{a^2}\right) - r^2 \quad -(29)$$

which is eq. (17), Q.E.D.

It has been shown that the metric (1) gives the Einstein energy equation (4) and the free particle hamiltonian (19). So the theory is rigorously self consistent.

Therefore the free particle hamiltonian (19) gives the orbit (17) or (29), even though there is no potential energy. The orbit is:

$$\frac{d\phi}{dr} = \frac{1}{r^2}\left(\frac{1}{b^2}-\frac{1}{a^2}-\frac{1}{r^2}\right)^{-1/2} \quad -(30)$$

So:

$$\phi = \int \frac{1}{r^2}\left(\frac{1}{b^2} - \frac{1}{a^2} - \frac{1}{r^2}\right)^{-1/2} dr \quad -(31)$$

Here:

$$a = \frac{L}{mc}, \quad b = \frac{Lc}{E} \quad -(32)$$

are constants. Here:

$$E = \gamma mc^2, \quad L = \gamma m r^2 \dot{\phi} \quad -(33)$$

and

$$\frac{dE}{dt} = 0, \quad \frac{dL}{dt} = 0 \quad -(34)$$

because E and L are constants of motion.

In the non-relativistic limit the orbit is:

$$\phi_0 = \int \frac{1}{r^2}\left(\frac{1}{b_0^2} - \frac{1}{a_0^2} - \frac{1}{r^2}\right)^{-1/2} dr \quad -(35)$$

where

$$a_0 = \frac{L_0}{mc}, \quad b_0 = \frac{L_0 c}{E_0} \quad -(36)$$

and

$$E_0 = \frac{1}{2} m v_N^2 \quad -(37)$$

$$L_0 = m r^2 \dot{\phi}_0 \quad -(38)$$

The Newtonian free particle hamiltonian is:

$$H_0 = E_0 = \frac{1}{2} m v_N^2 \quad -(39)$$

So the orbits (31) and (35) are consequences of the plane polar coordinate system. They can be worked out with computer algebra.

The orbit (31) corresponds to the equations:

$$\gamma^3 \frac{dv}{dt} = 0 \quad - (40)$$

and

$$\frac{dL}{dt} = 0 \quad - (41)$$

where

$$L = \gamma m r^2 \dot{\phi} \quad - (42)$$

This gives a check on the numerical procedure used to solve eqs. (40) and (41). The simultaneous solution of eqs. (40) and (41) must give eq. (31).

Having checked the code in this way it can be applied to:

$$\gamma^3 \frac{dv}{dt} = -\frac{MG}{r^2} \quad - (43)$$

and

$$\frac{dL}{dt} = 0 \quad - (44)$$

Eq. (31) can be integrated by denoting:

$$A := \frac{1}{b^2} - \frac{1}{a^2} \quad - (45)$$

so

$$\phi = \int \frac{1}{r^2} \left(A - \frac{1}{r^2} \right)^{-1/2} dr \quad - (46)$$

The Wolfram integrator gives:

$$\phi = -\tan^{-1} \left(\frac{1}{(Ar^2 - 1)^{1/2}} \right) \quad - (47)$$

so

$$\tan \phi = -\frac{1}{(Ar^2 - 1)^{1/2}} \quad - (48)$$

Therefore: $$A r^2 - 1 = \frac{1}{\tan^2\phi} \quad -(49)$$

and $$r^2 = \frac{1}{A}\left(\frac{1}{\tan^2\phi} - 1\right) \quad -(50)$$

$$= \left(\frac{1}{b^2} - \frac{1}{a^2}\right)^{-1}\left(\frac{1}{\tan^2\phi} - 1\right) \quad -(51)$$

Here, $$\left(\frac{1}{b^2} - \frac{1}{a^2}\right)^{-1} = \left(\frac{E^2}{L^2c^2} - \frac{m^2c^2}{L^2}\right)^{-1}$$

$$= \frac{L^2c^2}{E^2 - m^2c^4} \quad -(52)$$

$$= \frac{\gamma^2 m^2 r^4 \dot{\phi}^2 c^2}{(\gamma^2-1) m^2 c^4}$$

So $$r^2 = \left(\frac{L^2c^2}{E^2 - m^2c^4}\right)\left(\frac{1}{\tan^2\phi} - 1\right) \quad -(53)$$

Therefore the free particle moves along this trajectory.

Using $$E^2 = \gamma^2 m^2 c^4 \quad -(54)$$

it is found ~~that~~:

$$\boxed{r^2 = \frac{L^2}{c^2 m^2(\gamma^2-1)}\left(\frac{1}{\tan^2\phi} - 1\right)} \quad -(55)$$

As the non-relativistic limit is approached:

8)

$$\gamma^2 - 1 \rightarrow \left(1 - \frac{v^2}{c^2}\right)^{-1} - 1$$

$$\sim \frac{v^2}{c^2} \quad -(56)$$

So when

$$v << c \quad -(57)$$

it follows that:

$$r^2 \longrightarrow \frac{L_0^2}{m^2 v^2}\left(\frac{1}{\tan^2\phi} - 1\right) \quad -(56)$$

where

$$L_0 = m r^2 \dot{\phi} \quad -(57)$$

The non-relativistic angular momentum is therefore:

$$L_0 = m r v \left(\frac{1}{\tan^2\phi} - 1\right)^{-1/2} \quad -(57)$$

$$= m r^2 \dot{\phi}$$

Therefore in this limit:

$$v = r\dot{\phi}\left(\frac{1}{\tan^2\phi} - 1\right)^{1/2} \quad -(58)$$

In the next paper (UFT415), the changes made to this theory by the theory will be discussed and developed.