444(7): Relativistic Spin Connection due to Frame Rotation

In the observer frame the two relevant simultaneous equations are:

$$F = m\gamma^3(\ddot{r} - r\dot{\phi}^2) = -\frac{mMG}{r^2} \quad -(1)$$

and

$$\frac{d}{dt}(\gamma m r^2 \dot{\phi}) = \frac{dL}{dt} = 0 \quad -(2)$$

respectively the relativistic Leibniz equation and the conservation of relativistic angular momentum. The Lorentz factor is:

$$\gamma = \left(1 - \frac{1}{c^2}(\dot{r}^2 + r^2\dot{\phi}^2)\right)^{-1/2} \quad -(3)$$

It is assumed that the frame transform:

$$\phi' = \phi + \omega_1 t \quad -(4)$$

produces the relativistic spin connection and vacuum force.

From eq. (4):

$$\dot{\phi}' = \dot{\phi} + \omega_1 + t\frac{d\omega_1}{dt} \quad -(5)$$

in the frame (r, ϕ'). In this frame:

$$F = m\gamma'^3(\ddot{r} - r\dot{\phi}'^2) = -\frac{mMG}{r^2} \quad -(6)$$

$$L = \gamma' m r^2 \dot{\phi}' \quad -(7)$$

and

$$\frac{dL}{dt} = 0. \quad -(8)$$

So

$$F = m\gamma'^3\left(\ddot{r} - r\left(\dot{\phi} + \omega_1 + t\frac{d\omega_1}{dt}\right)^2\right) = -\frac{mMG}{r^2} \quad -(9)$$

Consider

$$\gamma^3 = \left(1 - \frac{1}{c^2}(\dot{r}^2 + r^2\dot{\phi}^2)\right)^{-3/2} \quad -(10)$$

and

$$\gamma'^3 = \left(1 - \frac{1}{c^2}(\dot{r}^2 + r^2\dot{\phi}'^2)\right)^{-3/2} \quad -(11)$$

It follows that:

$$\frac{1}{\gamma'^2} - \frac{1}{\gamma^2} = \frac{r^2}{c^2}\left(\left(\dot{\phi} + \omega_1 + t\frac{d\omega_1}{dt}\right)^2 - \dot{\phi}^2\right)$$

$$= \frac{r^2}{c^2}\left(\omega_1 + t\frac{d\omega_1}{dt}\right)\left(\omega_1 + t\frac{d\omega_1}{dt} + 2\omega\right)$$

$$:= A \quad -(12)$$

and that:

$$\gamma'^{-2} = \gamma^{-2} + A \quad -(13)$$

so

$$\gamma'^2 = \frac{\gamma^2}{1+\gamma^2 A} \quad -(14)$$

and

$$\gamma'^3 = \frac{\gamma^3}{(1+\gamma^2 A)^{3/2}} \quad -(15)$$

It follows that:

$$F = \frac{m\gamma^3}{(1+\gamma^2 A)^{3/2}}\left(\ddot{r} - r\dot{\phi}^2 - r\left(\omega_1 + t\frac{d\omega_1}{dt}\right)\left(\omega_1 + t\frac{d\omega_1}{dt} + 2\omega\right)\right)$$

$$= -\frac{mMG}{r^2} \quad -(16)$$

so

$$m\gamma^3(\ddot{r} - r\dot{\phi}^2) = -\frac{mMG}{r^2}(1+\gamma^2 A)^{3/2} + \frac{m\gamma^3 c^2 A}{r}$$

$$:= -\frac{mMG}{r^2} + \Omega_r \Phi \quad -(17)$$

here Ω_r is the relativistic spin connection and where:

$$\Phi = -\frac{MG}{r} \quad -(18)$$

3) It follows that:

$$\Omega_r = \frac{1}{r}\left(1+\gamma^2 A\right)^{3/2} - \frac{c^2}{MG}\gamma^3 A \qquad -(18)$$

where:

$$A = \frac{r^2}{c^2}\left(\omega_1 + t\frac{d\omega_1}{dt}\right)\left(\omega_1 + t\frac{d\omega_1}{dt} + 2\omega\right) \qquad -(19)$$

and

$$\gamma = \left(1 - \frac{1}{c^2}\left(\dot{r}^2 + r^2\dot{\phi}^2\right)\right)^{-1/2} \qquad -(20)$$

The two simultaneous equations to be solved are:

$$F = m\gamma^3\left(\ddot{r} - r\dot{\phi}^2\right) = -\frac{mMG}{r^2} + \Omega_r \Phi \qquad -(21)$$

and

$$\frac{dL}{dt} = \frac{d}{dt}\left(\gamma m r^2\left(\dot{\phi} + \phi_1 + t\frac{d\phi_1}{dt}\right)\right) = 0 \qquad -(22)$$