413(7): Cartan Torsion and Force from the Spin Connection.

In note 413(5) it was shown that the spin connection is:

$$\Omega_r = -\frac{L}{mMG}\left(\omega+\omega_1+t\frac{d\omega_1}{dt}\right)\left(\omega_1+t\frac{d\omega_1}{dt}\right)\left(\omega_1+t\frac{d\omega_1}{dt}+2\omega\right) \quad -(1)$$

and that this results from the frame rotation:

$$\phi' = \phi + \omega_1 t. \quad -(2)$$

The Cartan torsion associated with the spin connection (1) results in the acceleration due to gravity:

$$g_r = -\frac{MG}{r^2} - \frac{MG}{r}\Omega_r \quad -(3)$$

and the gravitational force:

$$F_r = m g_r = -\frac{mMG}{r^2} - \frac{mMG}{r}\Omega_r. \quad -(4)$$

The vacuum force is:

$$F_r(vac) = -\frac{mMG}{r}\Omega_r \quad -(5)$$

$$= \frac{L}{r}\left(\omega+\omega_1+t\frac{d\omega_1}{dt}\right)\left(\omega_1+t\frac{d\omega_1}{dt}\right)\left(\omega_1+t\frac{d\omega_1}{dt}+2\omega\right).$$

So the complete acceleration due to gravity results in the total force

$$F_r = m g_r = -\frac{mMG}{r^2} + F_r(vac) \quad -(6)$$

The total force is developed as in Lamb shift theory as being due to vacuum fluctuations δr. So

$$F_r(r+\delta r) = F(r) + F(vac) \quad -(7)$$

As in Note 413(2):

$$F_r(\text{vac}) = \frac{1}{6}\langle \delta \underline{r} \cdot \delta \underline{r} \rangle \nabla^2 F \quad -(8)$$

$$= -\frac{2}{3} m M G \frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^4} \quad -(9)$$

So the total force is:

$$F_r = -\frac{mMG}{r^2}\left(1 + \frac{2}{3}\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^2}\right) \quad -(10)$$

and the spin connection is:

$$\Omega_r = \frac{2}{3}\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^3} \quad -(11)$$

It follows from eqs. (1) and (11) that:

$$|\Omega_r| = \frac{2}{3}\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^3}$$

$$= \frac{L}{mMG}\left(\omega + \omega_1 + t\frac{d\omega_1}{dt}\right)\left(\omega_1 + t\frac{d\omega_1}{dt}\right)\left(\omega_1 + t\frac{d\omega_1}{dt} + 2\omega\right) \quad -(12)$$

In the limit: $\omega_1 \rightarrow 0 \quad -(13)$

it follows that: $|\Omega_r| \rightarrow 0 \quad -(14)$

self consistently. The orbit from the spin connection $|\Omega_r|$ is in general:

$$r = \frac{\alpha}{1 + \epsilon \cos(\phi + \omega_1 t)} \quad -(15)$$

and is a precessing ellipse. The precession is:

$$\Delta \phi = \omega_1 T \quad -(16)$$