HB(6): Note on Orbital Shrinkage.

The precessing orbit is:

$$r = \frac{\alpha}{1+\epsilon\cos(\phi+\omega_1 t)} \quad -(1)$$

where

$$\alpha = \frac{L^2}{m^2 MG} \quad -(2)$$

so:

$$r = \frac{L^2}{m^2 MG}\left(\frac{1}{1+\epsilon\cos(\phi+\omega_1 t)}\right) \quad -(3)$$

where:

$$L^2 = m^2 r^4\left(\omega+\omega_1+t\frac{d\omega_1}{dt}\right)^2 \quad -(4)$$

It follows that:

$$r = \frac{L^2 r^4}{MG}\frac{\left(\omega+\omega_1+t\frac{d\omega_1}{dt}\right)^2}{1+\epsilon\cos(\phi+\omega_1 t)} \quad -(5)$$

and

$$\boxed{r^3 = MG\frac{(1+\epsilon\cos(\phi+\omega_1 t))}{\left(\omega+\omega_1+t\frac{d\omega_1}{dt}\right)^2}} \quad -(6)$$

In which:

$$-1 \leq \cos(\phi+\omega_1 t) \leq 1 \quad -(7)$$

The angular velocity ω is defined as:

$$\omega = \frac{L_0}{mr^2} \quad -(8)$$

where L_0 is the constant angular momentum in the absence of frame rotation. It follows that:

$$L = mr^2\left(\omega+\omega_1+t\frac{d\omega_1}{dt}\right) \quad -(9)$$

$$= mr^2\left(\frac{L_0}{mr^2} + \omega_1 + t\frac{d\omega_1}{dt}\right)$$

So:

$$L = L_0 + mr^2\left(\omega_1 + t\left(\frac{d\omega_1}{dt}\right)\right) \quad -(10)$$

and:

$$\boxed{r^2 = \frac{L - L_0}{m\left(\omega_1 + t\frac{d\omega_1}{dt}\right)}} \quad -(11)$$

It follows that:

$$\boxed{r \xrightarrow[t \to \infty]{} 0} \quad -(12)$$

Q.E.D.

Therefore the orbit (1) shrinks as t increases.

From eqns (2) and (10) the half right latitude is:

$$\alpha = \frac{\left(L_0 + mr^2\left(\omega_1 + t\frac{d\omega_1}{dt}\right)\right)^2}{m^2 \underline{M} G} \quad -(13)$$

$$= \text{constant of motion}$$

so as t increases r must decrease to keep α constant, Q.E.D. Therefore:

) Find r from eqn (11) and plot it against t for given ω_1, $d\omega_1/dt$ and $L - L_0$.

) Use

$$r^2 = \frac{L_1}{\omega_1 + t\frac{d\omega_1}{dt}} = \left(\frac{\alpha}{1 + \epsilon\cos(\phi + \omega_1 t)}\right)^2 \quad -(14)$$

with

$$L = L_0 + L_1 \quad -(15)$$