413(3): Simplification of the Orbital Shrinkage Theory

Starting with the expression:

$$\frac{dr}{dt} = -\frac{r}{2}\frac{\frac{d}{dt}\left(\omega+\omega_1+t\frac{d\omega_1}{dt}\right)}{\omega+\omega_1+t\frac{d\omega_1}{dt}} \quad -(1)$$

use the rigorous definition:

$$\omega = \frac{L}{mr^2} \quad -(2)$$

so

$$\frac{d\omega}{dt} = \frac{d\omega}{dr}\frac{dr}{dt} = -\frac{2L}{mr^3}\frac{dr}{dt} \quad -(3)$$

It follows that:

$$\left(\omega+\omega_1+t\frac{d\omega_1}{dt}\right)\frac{dr}{dt} = \frac{L}{mr^2}\frac{dr}{dt} - \frac{r}{2}\left(\frac{d\omega_1}{dt}+\frac{d\omega_1}{dt}+t\frac{d^2\omega_1}{dt^2}\right) \quad -(4)$$

so:

$$\frac{dr}{dt} = -\frac{r}{2}\frac{\left(2\frac{d\omega_1}{dt}+t\frac{d^2\omega_1}{dt^2}\right)}{\omega_1+t\frac{d\omega_1}{dt}} \quad -(5)$$

in which:

$$r = \frac{\alpha'}{1+\epsilon'\cos\phi'} \quad -(6)$$

and

$$\phi' = \phi + \omega_1 t \quad -(7)$$

In this expression, α' and ϵ' are constants of motion, so can be coded in as constants.

As in previous work, the orbital

precession is:

$$\Delta\phi = \omega_1 T \quad -(8)$$

where T is the time taken for one orbit. The orbit is:

$$r = \frac{\alpha'}{1+\epsilon'\cos(\phi+\omega_1 t)} \quad -(9)$$

Suggested Graphics

1) Plot dr/dt for given $d\omega_1/dt$ and for given $d^2\omega_1/dt^2$, and ω_1. Fit this to data from the Hulse Taylor binary pulsar using a least mean squares curve fitting program. Alternatively a model of the time dependence of $\omega_1(t)$ can be chosen, for example:

$$\omega_1(t) = \omega_1(0)\exp(-\alpha t), \quad -(10)$$

so

$$\frac{d\omega_1}{dt} = -\alpha\,\omega_1(0)\exp(-\alpha t) \quad -(11)$$

and

$$\frac{d^2\omega_1}{dt^2} = \alpha^2\omega_1(0)\exp(-\alpha t) \quad -(12)$$

i.e.

$$\frac{d\omega_1}{dt} = -\alpha\,\omega_1 \quad -(13)$$

and

$$\frac{d^2\omega_1}{dt^2} = \alpha^2\omega_1 \quad -(14)$$

2) Plot the orbit (9) for a given ω_1.