413(2): Vacuum Force and Vacuum Fluctuation

In the rotating frame, the linear velocity in general is:

$$\underline{v}' = \dot{r}'\,\underline{e}_r' + r'\dot{\phi}'\,\underline{e}_\phi' \quad -(1)$$

and the acceleration is:

$$\underline{a}' = (\ddot{r}' - r'\dot{\phi}'^2)\,\underline{e}_r' + (r'\ddot{\phi}' + 2\dot{r}'\dot{\phi}')\,\underline{e}_\phi' \quad -(2)$$

where the primes denote rotating frame. In this plane polar system the unit vectors in the rotating frame are:

$$\underline{e}_r' = \underline{i}\cos(\phi + \omega_1 t) + \underline{j}\sin(\phi + \omega_1 t) \quad -(3)$$

and

$$\underline{e}_\phi' = -\underline{i}\sin(\phi + \omega_1 t) + \underline{j}\cos(\phi + \omega_1 t) \quad -(4)$$

The total force is:

$$\underline{F}' = m\underline{a}' = -\frac{\partial U}{\partial r}\underline{e}_r' + \underline{\Omega}'U \quad -(5)$$

where $\underline{\Omega}'$ is the vector spin connection. The primes denote a rotating frame defined by:

$$\phi' = \phi + \omega_1 t \quad -(6)$$

Eq. (5) can be written as:

$$\underline{F}'(\underline{r} + \delta\underline{r}) = \underline{F}'(\underline{r}) + \underline{F}'(\text{vac}) \quad -(7)$$

where $\delta\underline{r}$ is the vacuum fluctuation of $\underline{r}$ and where $\underline{F}'(\text{vac})$ is the vacuum force:

$$\underline{F}'(\text{vac}) = \underline{F}'(\underline{r} + \delta\underline{r}) - \underline{F}'(\underline{r}) \quad -(8)$$

Therefore:

$$\underline{F}(\underline{r} + \delta\underline{r}) = m\underline{a}' \quad -(9)$$

$$\underline{F}(\underline{r}) = -\frac{\partial U}{\partial r}\underline{e}_r' \quad -(10)$$

and

$$\underline{\Omega}'U = \underline{F}(\text{vac}) \quad -(11)$$

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If it is assumed that:

$$\underline{\Omega}' = \Omega_r' \underline{e}_r' \quad -(12)$$

then eq. (7) can be written in its component form:

$$\frac{F_r'}{m} = \ddot{r}' - r\dot{\phi}'^2 = -\frac{mG}{r^2} + \Omega_r' \Pi \quad -(13)$$

and

$$\frac{F_\phi'}{m} = r\ddot{\phi}' + 2\dot{r}\dot{\phi}' = 0 \quad -(14)$$

in which:

$$\dot{\phi}' = \omega + \omega_1 + t\frac{d\omega_1}{dt} \quad -(15)$$

$$\ddot{\phi}' = \frac{d\omega}{dt} + 2\frac{d\omega_1}{dt} + t\frac{d^2\omega_1}{dt^2} \quad -(16)$$

Eq. (13) can be written as:

$$F_r'(\underline{r} + \delta\underline{r}) = F_r'(\underline{r}) + F_r'(\text{vac}) \quad -(17)$$

Using a Tensor Taylor series expansion to first order:

$$F_r'(\text{vac}) = \Delta F_r' = F(r + \delta r) - F(r)$$

$$= \frac{1}{6}\langle \delta\underline{r} \cdot \delta\underline{r} \rangle \nabla^2 F \quad -(18)$$

where

$$F = -\frac{mMG}{r^2} \quad -(19)$$

Therefore the vacuum force and the spin connection are produced by frame rotation.

So:

$$F_r'(\text{vac}) = -\frac{2}{3} mMG \frac{\langle \delta r \cdot \delta r \rangle}{r^4} \quad -(20)$$

Therefore the total force of attraction between

3) m and $\underline{M}$ is:

$$\boxed{F_r' = mg'(\underline{r}+\delta\underline{r}) = -\frac{mMG}{r^2}\left(1+\frac{\langle \delta\underline{r}\cdot\delta\underline{r}\rangle}{r^2}\right)} \quad -(21)$$

This force is:

$$F_r' = m(\ddot{r} - r\dot{\phi}'^2) \quad -(22)$$

$$= -\frac{mMG}{r^2}\left(1 + r\Omega_r'\right)$$

so

$$\boxed{\Omega_r' = \frac{\langle \delta\underline{r}\cdot\delta\underline{r}\rangle}{r^3}} \quad -(23)$$

These equations produce the orbit:

$$r = \frac{\alpha'}{1+\epsilon'\cos\phi'} \quad -(24)$$

and the precession:

$$\Delta\phi = \omega_1 T \quad -(25)$$

The rotation of the frame is due to spacetime torsion, and the torsion produces the angular velocity ω_1 of frame rotation, the vacuum force, the isotropic fluctuation $\langle \delta\underline{r}\cdot\delta\underline{r}\rangle$ and the spin connection and orbital precession.