410(7): Universal Law of Precession Applied to the S2 Star System

As in UFT375, the S2 star system is characterized by:

$$
\begin{aligned}
M &= 7.956 \times 10^{36} \text{ kg} \\
G &= 6.67408 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \\
c &= 2.99792458 \times 10^{8} \text{ m s}^{-1} \\
a &= 1.4253 \times 10^{14} \text{ m} \\
\epsilon &= 0.8831 \\
T &= 15.56 \text{ earth years.} \\
\Delta\phi &= 3.549 \times 10^{-3} \text{ radians per S2 orbit}
\end{aligned}
\quad -(1)
$$

Here M is a very large mass at the centre of the Milky Way galaxy, G is Newton's constant, c the speed of light, a the semi major axis of the S2 orbit, ϵ its eccentricity, T is the time taken for one orbit, and $\Delta\phi$ is the orbital precession.

Therefore the orbital precession per earth year is:

$$\Delta\phi = \frac{3.549}{15.56} \times 10^{-3} = 2.281 \times 10^{-4} \text{ radians} \quad -(2)$$

At the perihelion, or distance of closest approach of S2 to M:

$$r = a(1-\epsilon) = 1.67 \times 10^{13} \text{ m} \quad -(3)$$

so the Newtonian velocity at the perihelion is

$$V_N^2 = \frac{MG}{a}\left(\frac{1}{1-\epsilon} - 1\right) \quad -(4)$$

$$= 60.02 \times 10^{12} \text{ m}^2 \text{ s}^{-2}$$

The universal law of precession is:

$$\Delta\phi = \frac{2\pi}{c^2} v^2 \quad -(5)$$

where v^2 is defined by the rotation of the ECE2 line element. So

$$v^2 = \frac{c^2}{2\pi}\Delta\phi = 4.022 \times 10^{12} \text{ m}^2 \text{ s}^{-2} \quad -(6)$$

a) Note carefully that in the S2 star system:

$$\boxed{v^2 < v_N^2} \quad -(7)$$

This result is explained by rotating the line element with

$$\phi' = \phi - \omega t \quad -(8)$$

so the rotated line element is:

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi'^2$$
$$= c^2 dt^2 - dr^2 - r^2 d\phi^2 + 2\omega r^2 d\phi dt - \omega^2 r^2 dt^2$$
$$-(9)$$

Now use:

$$v_\theta = \omega r \quad -(10)$$
$$dr^2 + r^2 d\phi^2 = v_N^2 dt^2 - (11)$$
$$d\phi = \omega dt \quad -(12)$$

to find that:

$$ds^2 = (c^2 - v_N^2 + v_\theta^2)\, dt^2 - (13)$$
$$= \left(1 - \frac{(v_N^2 - v_\theta^2)}{c^2}\right) c^2 dt^2$$
$$:= \left(1 - \frac{v^2}{c^2}\right) c^2 dt^2$$

where

$$\boxed{v^2 = v_N^2 - v_\theta^2} \quad -(14)$$

In previous work it has been shown that the result for

$$\phi' = \phi + \omega t \quad -(15)$$

is

$$v^2 = v_N^2 + 3v_\theta^2 \quad -(16)$$

in which

$$\boxed{v^2 > v_N^2} \quad -(17)$$

3)

The universal law of precession is therefore:

$$\Delta\phi = \frac{2\pi}{c^2}\left(v_N^2 + 3\omega^2 r^2\right) \quad -(18)$$

for

$$\phi' = \phi + \omega t \quad -(19)$$

and

$$\Delta\phi = \frac{2\pi}{c^2}\left(v_N^2 - \omega^2 r^2\right) \quad -(20)$$

for

$$\phi' = \phi - \omega t \quad -(21)$$

For the S2 star system:

$$v_\theta^2 = v_N^2 - v^2$$
$$= 56.02 \times 10^{12}\ m^2 s^{-2} \quad -(22)$$

so

$$\omega_- = \frac{v_\theta}{r} \quad -(23)$$

where ω_- denotes the angular velocity from eq. (8) and ω_+ denotes the angular velocity from eq. (15).

At the perihelion it is found that

$$\boxed{\omega_- = 4.48 \times 10^{-7} \text{ radians}} \quad -(24)$$

Table 1

System	Precession (radians per earth year)	Angular Velocity of Frame (radians per second)
S2 Star	2.281×10^{-4}	$\omega_- = 4.48 \times 10^{-7}$
Hulse Taylor Binary Pulsar	0.0738	$\omega_+ = 5.65 \times 10^{-3}$
Total Precession of Mercury	2.778×10^{-4}	$\omega_+ = 1.99 \times 10^{-5}$