

# +09(2) : Relation between Thomas velocity and Newtonian velocity

Consider the FEE2 covariant metric:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 \quad (1)$$

in plane polar coordinates  $(r, \phi)$ . The Thomas velocity  $v_T$  is defined by the rotation:

$$\phi' = \phi + \omega t \quad (2)$$

also

$$\omega r = v_T \quad (3)$$

This rotation leads to the Thomas precession:

$$\Delta \phi_T = 2\pi \beta \quad (4)$$

also

$$\beta = \left(1 - \frac{v_T^2}{c^2}\right)^{-1/2} - 1 \quad (5)$$

The Lorentz factor  $\gamma$  is defined by the unrotated metric (1):

$$ds^2 = c^2 d\tau^2 = (c^2 - v_N^2) dt^2 \quad (6)$$

where  $v_N$  is the Newtonian velocity. So

$$\gamma = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} = \frac{dt}{d\tau} \quad (7)$$

From eqs. (1) and (6) the Newtonian velocity is defined

$$v_N^2 = \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\phi}{dt}\right)^2 \quad (8)$$

in the unrotated metric.

Under the condition:

$$v_T = v_N \quad (9)$$

it follows that:

$$2) \quad \beta = \gamma - 1 \quad - (10)$$

More generally:  $\beta \neq \gamma - 1 \quad - (11)$

Hypothesis Planetary precession is defined by the Thomas velocity  $v_T$ .

The precession of any planet is therefore:

$$\Delta \phi_T = 2\pi \left( \left( 1 - \frac{v_T^2}{c^2} \right)^{-1/2} - 1 \right) \quad - (12)$$

In the solar system:

$$\Delta \phi_T = 2\pi \left( \frac{1}{2} \frac{v_T^2}{c^2} \right) \quad - (13)$$

where

$$\beta_0 = \frac{1}{2} \frac{v_T^2}{c^2} \quad - (14)$$

is the Thomas half. Therefore the precession of any object in the solar system, or in general anywhere in the universe, can be described by eq. (12) and its low velocity limit, eq. (14).

Note carefully that the Newtonian eq. (8) does not produce a precessing orbit. It produces the conic section

$$r = \frac{d}{1 + \epsilon \cos \phi} \quad - (15)$$

where  $d$  is the half right distance and  $\epsilon$  the eccentricity. As is well known, eq. (15) produces:

$$v_H^2 = mG \left( \frac{2}{r} - \frac{1}{a} \right) = \frac{mG}{R_0} (1 + \epsilon) \quad - (16)$$

where

$$a = \frac{d}{1 - \epsilon^2} \quad - (17)$$

is the semi major axis and

3)

$$R_0 = \frac{d}{1+E} \quad (18)$$

in the perihelion or 1+E distance of closest approach. The Thomas and Newtonian velocities are different concepts, the former is relativistic and the latter is non-relativistic. The former produces planetary precession and the latter does not. The two velocities coincide only in the non-relativistic limit:

$$\frac{v_T^2}{c^2} = \frac{v_N^2}{c^2} \rightarrow 0 \quad (19)$$

The Thomas factor  $\beta$  and Thomas half  $\beta_0$  must always be defined by  $v_T$ , and not by  $v_N$ ! In the limit (19) for a Newtonian circular orbit,

$$v_T^2 = \omega^2 r^2 = v_N^2 = \frac{MG}{r} \quad (20)$$

so

$$\omega^2 = \frac{MG}{r^3} \quad (21)$$

This is Kepler's third law, obtained from the force equation

$$m(\ddot{r} - r\dot{\phi}^2) = -\frac{rMG}{r^2} \quad (22)$$

when:

$$\ddot{r} = 0, \quad \omega = \dot{\phi} \quad (23)$$

From (2) & (1)

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 (d\phi + \omega dt)^2 \quad (24)$$

so the Newtonian velocity is changed to:

$$\begin{aligned} v^2 &= \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\phi}{dt}\right)^2 + 2r^2 \omega \frac{d\phi}{dt} + r^2 \omega^2 \\ &= \left(\frac{dr}{dt}\right)^2 + r^2 \frac{(d\phi + \omega dt)^2}{dt^2} \quad (25) \end{aligned}$$

Therefore the frame rotation argument & Newtonian velocity follows:

$$v_i^2 = v_N^2 + v_T^2 + 2r^2 \omega \frac{d\phi}{dt} \quad (26)$$

It produces the planetary precession:

$$\Delta \phi_T = 2\pi \left( \left( 1 - \frac{v_T^2}{c^2} \right)^{-1/2} - 1 \right) \quad (27)$$

In the solar system,  $\Delta \phi_T$  is very tiny, and:

$$v_i \sim v_N \quad (28)$$

is an excellent approximation.

Interpretation of  $\beta$  and  $\beta_0$

Interpretation: The results of UFT408 are derived with the

$$\beta = \gamma - 1 = \left( 1 - \frac{v_N^2}{c^2} \right)^{-1/2} - 1 \quad (29)$$

$$\beta_0 = \frac{1}{2} \frac{v_N^2}{c^2} \quad (30)$$

Planetary precession is described with the Interpretation:

$$\Delta \phi_T = 2\pi \beta \quad (31)$$

$$\beta = \left( 1 - \frac{v_T^2}{c^2} \right)^{-1/2} - 1 \quad (32)$$

The Lorentz factor in the presence of Thomas rotation becomes:

$$\gamma_i = \left( 1 - \frac{v_i^2}{c^2} \right)^{-1/2} \quad (33)$$

Since planetary precession is always observed, the Lorentz factor is generalized to Eq. (33).

Therefore the fundamental E(E) covariant quantities  
 all generalized, by replacing  $\gamma$  by  $\gamma_1$ . For example the  
 kinetic velocity is:

$$\underline{v} = \gamma_1 \underline{v} \cdot 1 \quad (34)$$

$$v^2 = \frac{v_1^2}{1 - \frac{v_1^2}{c^2}} \quad (35)$$

$$v_1^2 = \frac{v^2 \frac{c^2}{1 + \frac{v^2}{c^2}}}{1 + \frac{v^2}{c^2}} \quad (36)$$

The Lorentz factor is given by:

$$\gamma_1 = \left(1 + \frac{v^2}{c^2}\right)^{1/2} = \left(1 - \frac{v_1^2}{c^2}\right)^{-1/2} \quad (37)$$

The Newtonian light deflection due to gravitation becomes:

$$\Delta\phi = \frac{2mb}{R_0 v_1^2} \quad (38)$$

The velocity of light is  $c$ , so:

$$v \rightarrow c \quad (39)$$

$$v_1^2 \rightarrow \frac{c^2}{2} \quad (40)$$

and giving the exactly correct experimental result:

$$\Delta\phi = \frac{4mb}{R_0 c^2} \quad (41)$$

The Hamiltonian becomes:

$$H = \gamma_1 mc^2 + U \quad (42)$$

The Lagrangian becomes:

$$\mathcal{L} = - \frac{mc^2}{\gamma_1} - U \quad (43)$$