Analytical orbital equation for ECE2 covariant precession

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3 Graphical and analytical calculations

We give some examples of relativistic orbit effects. For an analytical computation we first need an orbit $r(\phi)$ for which we use the non-relativistic elliptic orbit

$$r(\phi) = \frac{\alpha}{1 + \epsilon \cos(\phi)} \tag{62}$$

where α is the semi latus rectum and the semi major axis is

$$a = \frac{\alpha}{1 + \epsilon^2} \tag{63}$$

with eccentricity ϵ. The corresponding velocity of the orbiting mass is

$$v^2 = MG \left(\frac{2}{r} - \frac{1}{a} \right). \tag{64}$$

According to Eqs. (2-11) in section 2, the relativistic gamma factor is

$$\gamma = \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \tag{65}$$

and the modulus of the spin connection is

$$\omega = \frac{1}{r} \left(1 - \left(1 - \frac{v^2}{c^2} \right)^{3/2} \right). \tag{66}$$

This is connected with to the isotropic square of vacuum fluctuations via

$$\langle \delta \mathbf{r} \cdot \delta \mathbf{r} \rangle = \frac{3}{2} r^3 \omega \tag{67}$$

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whose modulus can be defined by

$$\langle \delta r \rangle = \sqrt{\langle \delta \mathbf{r} \cdot \delta \mathbf{r} \rangle} = \sqrt{\frac{3}{2} r^3 \omega}. \tag{68}$$

Finally the squared angular frequency of the vacuum fluctuation is

$$\Omega_0^2 = \frac{2}{3} \frac{MG}{r^4} \langle \delta r \rangle . \tag{69}$$

These quantities have been graphed in Figs. 1-3. All parameters have been set to unity except the velocity of light $c = 2$ and the eccentricity $\epsilon = 0.3$. As can be seen from Fig. 1, for this eccentricity the radius varies roughly between 0.7 and 1.4 units. Figs. 2 and 3 have been restricted to this range. The ratio v/c (Fig. 2) is highly relativistic, this range has been chosen to see clear graphical effects, although the true velocity will have significant deviations from the non-relativistic approximation used here. Correspondingly, the γ factor raises to 1.4 at perihelion.

The spin connection ω (Fig. 3) is largest at perihelion which is plausible. The fluctuation frequency Ω_0 parallels the spin connection quite precisely. The average fluctuation radius $\langle \delta r \rangle$ is only varying slightly. In total it can be seen that the central mass distorts the space around it.

The relativistic Binet equation (60) is not solvable analytically with x given by Eq. (58). Assuming a constant x leads to an effective change of the half right latitude α. This gives a change of ellipse dimensions but no precession. x must have a coordinate dependence to give such an effect.

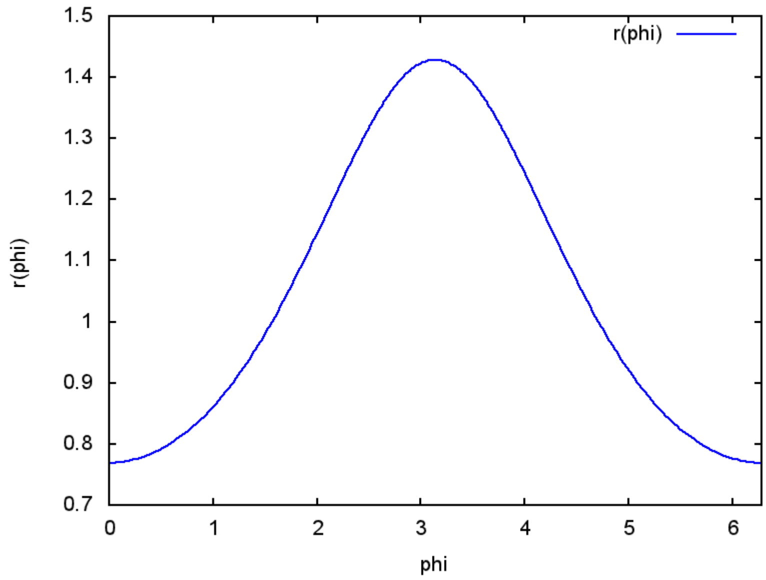

Figure 1: Elliptic orbit $r(\phi)$ for an orbiting mass.

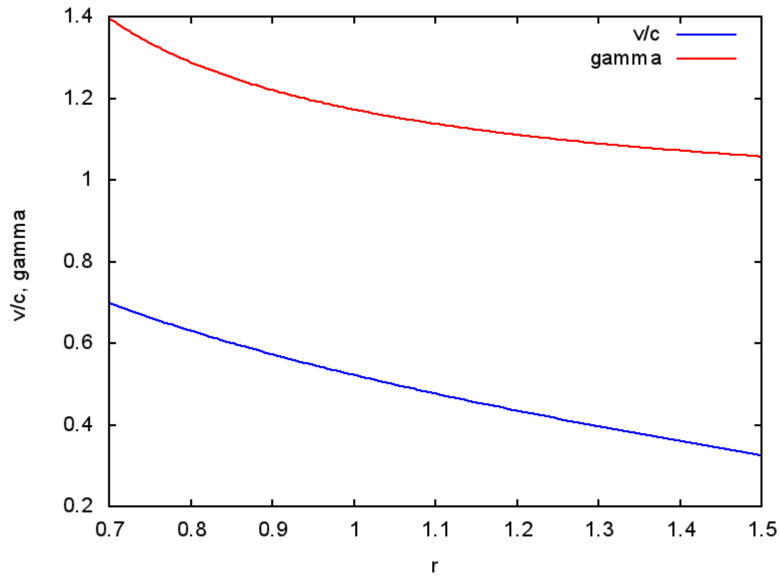

Figure 2: Ratio v/c and relativistic gamma factor.

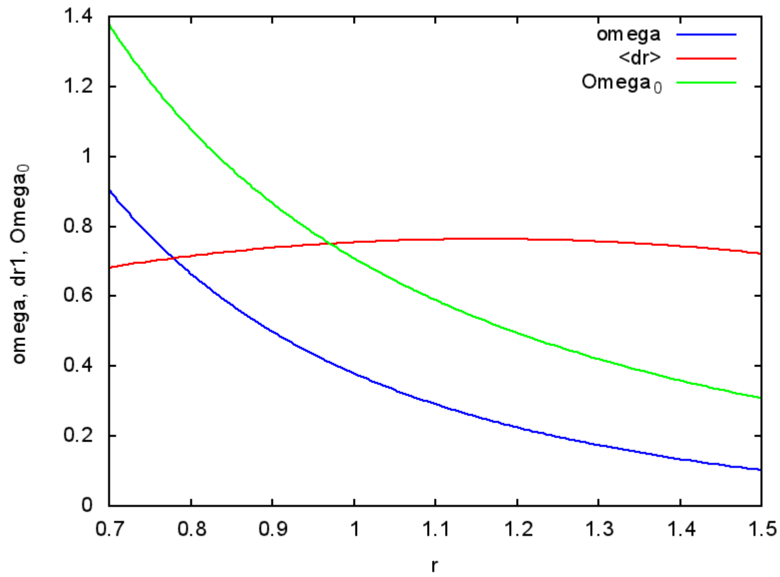

Figure 3: Spin connection ω, averaged fluctuation radius $\langle \delta r \rangle$ and fluctuation frequency Ω_0.