Calculation of Precession due to a Rotating Object

As noted, the force per unit mass $f$ is:

$$ f = -\nabla \phi - \frac{\partial \mathbf{a}}{\partial t} \quad (1) $$

$$ = -\nabla \phi + \omega \times \mathbf{r} \quad (2) $$

so the vacuum force is:

$$ f_{(\text{vac})} = \omega \times \mathbf{r} - \frac{\partial \mathbf{a}}{\partial t} \quad (3) $$

The rotational angle is:

$$ \psi = \pi \left( 3 + \frac{\partial f^{(1)}}{\partial r} \right)^{-1/2} - (4) $$

As in UFT03, the total rotational angle, for example:

$$ \Delta \phi = \frac{1}{2} \left( \frac{\omega}{r} - \frac{\partial \omega}{\partial r} \right) \quad (5) $$

and

$$ \omega = |\omega| \quad (6) $$

$$ -\frac{\partial \mathbf{a}}{\partial t} = \omega \times \mathbf{r} \quad (7) $$
The magnitude of the spin connection is:

$$\omega = \frac{2}{3} \left( \frac{1}{\Delta \phi} \left( \frac{<S_x \cdot S_x >}{r^3} - \frac{1}{3r} \frac{d}{dr} <S_z \cdot S_z > r \right) \right)$$

where $<S_x \cdot S_x >$ is the isotropically averaged fluctuation of the vacuum ( factored out already). Therefore:

$$\Delta \phi = \frac{4}{3} \left( \frac{<S_x \cdot S_x >}{r^3} - \frac{1}{3r} \frac{d}{dr} <S_z \cdot S_z > r \right)$$

From eq. (7):

$$\omega \phi = -\frac{1}{2} |Q_{(Total)}|$$

In previous papers of the UFT series the laser-lightning effect was developed with the gravitomagnetic field:

$$\Omega = \frac{6}{c^2 r^3} \left( \frac{3m_g \cdot S_x}{r^2} - m_g \right) = \frac{6}{2c^2 r^3} \left( \frac{3L \cdot S_x}{r^2} - L \right)$$

where

$$m_g = \frac{1}{2} \frac{L}{r}$$

is the gravitomagnetic dipole moment and $L$ the angular momentum of the earth.

By definition:

$$\Omega = \nabla \times \Omega_{(Total)}$$

So:
\[
\Omega_{(\text{total})} = \frac{G - mg \times r}{c^2} = \frac{G}{2c^2} \frac{L \times r}{r^3} \quad -(14)
\]

Therefore:
\[
|\Omega_{(\text{total})}| = \frac{G}{2c^2} \frac{L \times r}{r^3} \quad -(15)
\]

By vector algebra:
\[
L \times r \cdot L \times r = L^2 r^2 - (L \cdot r)^2 \quad -(16)
\]
\[
= (L \cdot L)(r \cdot r) - (L \cdot r)(L \cdot r) \quad -(17)
\]

So:
\[
|\Omega_{(\text{total})}| = \frac{G}{2c^2} \left( L^2 r^2 - (L \cdot r)^2 \right)^{1/2} \quad -(17)
\]

So:
\[
\frac{d}{dt} |\Omega_{(\text{total})}| = \frac{G}{2c^2} \frac{d}{dt} \left( \frac{1}{r^3} \left( L^2 r^2 - (L \cdot r)^2 \right)^{1/2} \right) \quad -(18)
\]

In Cartesian coordinates:
\[
\mathbf{v} = \frac{d\mathbf{s}}{dt} \quad -(19)
\]

is orbital velocity. So
\[
\mathbf{v} = \frac{d\mathbf{s}}{dt} \quad -(20)
\]
If \( \mathcal{E}(\tau) = \frac{1}{r^2} \left( \frac{L}{2} - (L \cdot \tau)^2 \right)^{1/3} \), then

\[
\frac{d\mathcal{E}(\tau)}{dt} = \frac{dr}{dt} \frac{d\mathcal{E}(\tau)}{dr}.
\]

Then

\[
v \frac{d}{ds} \left( \frac{1}{r^2} \left( \frac{L}{2} - (L \cdot \tau)^2 \right)^{1/3} \right) - \frac{c}{2c^2} \]

and

\[
\frac{d}{dt} |\mathcal{E}(\text{total})| = 6v \frac{d\mathcal{E}(\tau)}{dr} - \frac{c}{2c^2}.
\]

From eqs. (10) and (23):

\[
\omega \phi = - \frac{6v}{2c^2} \frac{d\mathcal{E}(\tau)}{dr} - (24)
\]

where

\[
\phi = - \frac{\mathcal{E}(\tau)}{r} - (25)
\]

so

\[
\omega = - \frac{c v}{2c^2} \frac{d\mathcal{E}(\tau)}{dr} - (26)
\]

If the sign of \( \omega \) is reversed in eq. (1), then

\[
\mathcal{E}(\tau) = - \frac{1}{r} \phi - \omega \phi - (27)
\]

and

\[
\omega \phi = \frac{d}{dt} |\mathcal{E}(\text{total})| - (28)
\]

so

\[
\omega = \frac{c v}{2c^2} \frac{d\mathcal{E}(\tau)}{dr} - (29)
\]
The precession in radians per second is given by

$$\Delta \phi = \frac{r}{2} \left( \frac{c - \frac{2c}{r}}{r} \right) - (30)$$

In Gravitic Prede B for example, $r$ is the distance from the centre of the Earth to the spacecraft, and $V$ is its orbital period. Here, $M$ is the mass of the Earth and $c$ is the speed of light. The angular velocity of the Earth is taken from UFT 117 - UFT 119 and UFT 145:

$$L = 2 \frac{M r_E^2 \omega_E}{5}$$ - (31)

where $M$ is the mass of the Earth, $r_E$ is the Earth's radius, $\omega_E$ is its angular velocity.

$$M = 5.98 \times 10^24 \text{ kg}$$
$$r_E = 6.37 \times 10^6 \text{ m}$$
$$\omega_E = 7.292 \times 10^{-5} \text{ rad s}^{-1}$$ - (32)

For gravity Prede B:

$$r = 7.02 \times 10^6 \text{ m}$$ - (33)

On average, but varies in general. The constants are:

$$c = 2.998 \times 10^8 \text{ m s}^{-1}$$ - (34)
$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$ - (35)

Therefore $\Delta \phi$ can be evaluated with computer algebra. For a given $r$ and $V$, the result adjusted to give agreement with experiment.

The precession is experimentally very tiny, so the Newtonian theory is an excellent approximation for
\( r = \frac{d}{1 + \epsilon \cos \phi} \quad - (36) \)

and

\[ v^2 = \sqrt{6} \left( \frac{2}{r} - \frac{1}{a} \right) \quad - (37) \]

\[ a = \frac{d^2}{1 - \epsilon^2} \quad - (38) \]

Here \( d \) is the half right ascension, and \( \epsilon \) is the ellipticity of the orbit of Earth about B.

If \( \mathbf{L} \) is perpendicular to \( \mathbf{r} \), expressionally, the

\[ \mathbf{L} \cdot \mathbf{r} = 0 \quad - (39) \]

and

\[ \omega = \pm \frac{5v}{2Mc^5} \frac{d}{dr} \frac{\mathbf{L}}{r^3} \]

\[ = \pm \frac{vL}{2Mc^5} \quad - (40) \]

where \( L \) is the angular momentum in arc sec. The velocity in eqn. (40) is given by eqn. (37) and the angular momentum by eqn. (31). If it is assumed that

\[ \epsilon \gg 1 \quad - (41) \]

then:

\[ \omega \approx \frac{2}{5} \frac{\omega EV}{c^2} \quad - (42) \]

where the velocity of rotation of the Earth about its axis is:

\[ v \approx 4.60 \times 10^5 \text{ m/s} \quad - (43) \]

Hence the magnitude of the spin can be given as
\[ \omega \approx \frac{5}{2} \times \frac{7.292 \times 10^7}{2.998 \times 10^8} \times 4.60 \times 10^{-13} \ m^{-1} \]

\[ = 3.0 \times 10^{-13} \ m^{-1} \]  

From Eqs. (30) and (46) the precession is radians per second is:

\[ \Delta \phi = \frac{r}{2} \left( \frac{\omega}{r} - \frac{d\omega}{dr} \right) \]  

Taking the negative value of \( \omega \) in Eq. (46) then:

\[ \frac{\omega}{r} = -\frac{L}{r^3} \frac{v}{M c^2} \]  

and:

\[ \frac{d\omega}{dr} = \frac{L}{r^2} \frac{d}{dr} \left( \frac{v}{r^3} \right) \]

\[ = \frac{L}{r^3} \left( \frac{1}{r} \frac{dv}{dr} + \frac{v}{r} \frac{d}{dr} \left( \frac{1}{r^3} \right) \right) \]

\[ = \frac{L}{r^3} \left( -2 \frac{v}{r} + \frac{1}{r^2} \frac{dv}{dr} \right) \]

So:

\[ \Delta \phi = \frac{L r^2}{2 M c^2} \left( \frac{v}{r^3} + \frac{1}{r^2} \frac{dv}{dr} \right) \]

\[ = \frac{L}{2 M c^2} \left( \frac{v}{r} + \frac{dv}{dr} \right) \]  

If \( v \) is approximately constant:

\[ v \approx 4.60 \times 10^7 \ m/s \]  

\[ \omega \approx 3.0 \times 10^{-13} \ m^{-1} \]
\[ \frac{dv}{dr} = 0 \quad \text{(50)} \]

and

\[ \Delta \phi \sim \frac{1}{5} \frac{\omega e v r}{c^2} \quad \text{(51)} \]

\[ = 5.24 \times 10^{-13} \text{ rad per year} \]

2) Vectorial Method

This method is given in UFT 345, and gives

\[ \Delta \phi \sim 3.18 \times 10^{-13} \text{ rad per year} \]

The experimental result from Gravity Probe B

is claimed to be

\[ \Delta \phi \text{ (exp)} \sim 1.016 \times 10^{-13} \text{ rad per year} \quad \text{(52)} \]

An averaging method was used in UFT 345

to give precise agreement with experimental data.

The approximate result (51) from a vectorial method can be made exact using complex algebra,

and the averaging procedure used to give precise agreement with experimental data.