(6) Precession from the Apsidal Angle

(farside.ph.utexas.edu/teaching/336k)

For nearly circular orbits of low eccentricity, the apsidal angle is defined as

$$\phi = \pi \left(3 + r \frac{F'}{F} \right)^{-1/2} \quad -(1)$$

here

$$F' = \partial F / \partial r \quad -(2)$$

Eq. (1) gives a simple method of calculating the precession of the perihelion for a given force law, as developed in previous UFT papers. Fitzpatrick in the above reference considers the force law of the Einstein theory:

$$F = -\frac{mMG}{r^2} - \frac{3MGL^2}{mc^2 r^4} \quad -(3)$$

It is known that the force law (3) is obsolete, but is used in the first part of this note to illustrate the method.

From eq. (3)

$$\frac{\partial F}{\partial r} = \frac{2mMG}{r^3} + \frac{12MGL^2}{mc^2 r^5} \quad -(4)$$

so

$$r \frac{F'}{F} = -\frac{\left(2\frac{mMG}{r^2} + \frac{12MGL^2}{mc^2 r^4} \right)}{\frac{mMG}{r^2} + \frac{3MGL^2}{mc^2 r^4}} \quad -(5)$$

$$= -\frac{\left(\frac{2}{r^2} + 12 \frac{L^2}{m^2 c^2 r^4} \right)}{\frac{1}{r^2} + 3\frac{L^2}{m^2 c^2 r^4}}$$

$$= -2 \frac{\left(1 + \frac{6L^2}{m^2 c^2 r^2} \right)}{\left(1 + \frac{3L^2}{\cdots} \right)}$$

If $$\frac{3L^2}{m^2c^2r^2} << 1 \quad -(6)$$

eq:

$$\frac{rF'}{F} \sim -2\left(1+\frac{6L^2}{m^2c^2r^2}\right) \quad -(7)$$

and

$$\phi = \pi\left(3-2\left(1+\frac{6L^2}{m^2c^2r^2}\right)\right)^{-1/2}$$

$$= \pi\left(1-\frac{12L^2}{m^2c^2r^2}\right)^{-1/2}$$

$$\sim \pi\left(1+\frac{6L^2}{m^2c^2r^2}\right) \quad -(8)$$

At the perihelia:

$$r^2 = a^2(1-\epsilon)^2 \quad -(9)$$

where a is the semi major axis and ϵ the eccentricity

For an approximately circular orbit:

$$L^2 = m^2 MG\alpha \sim m^2 MGr \quad -(10)$$

So:

$$\phi = \pi\left(1+\frac{6MG}{c^2a(1-\epsilon)}\right) \quad -(11)$$

so

$$\Delta\phi = \frac{6\pi MG}{c^2a(1-\epsilon)} \quad -(12)$$

$$+ \cdots\cdots$$

The same result as given by Marion and

) Thornton, eq. (7.84c) of the Third edition, is:

$$\Delta\phi \sim \frac{6\pi MG}{ac^2(1-\epsilon^2)} \quad -(13)$$

using a method of successive approximations that has been criticized in previous UFT papers. So the two methods give different results, but the spin connection method is much simpler. It is pointless to claim that the Einstein theory is precise, because precessions in the solar system are very small and heavily influenced by motion of other planets.

precession from the ECE2 Force Equation

Consider the ECE2 force equation:

$$\underline{F} = -\underline{\nabla}\phi + \underline{\omega}\phi \quad -(14)$$

in which

$$\omega = \frac{2}{3}\frac{\langle \delta\underline{r}\cdot\delta\underline{r}\rangle}{r^3} \quad -(15)$$

is the magnitude of the spin connection. The latter originates in the vacuum fluctuation:

$$\delta\underline{r} = \delta\underline{r}(0)\exp(-i\Omega t) \quad -(16)$$

The vacuum force is

$$\underline{F}(\text{vac}) = \underline{\omega}\phi \quad -(17)$$

and a tensor Taylor expansion gives the isotropically averaged magnitude of the vacuum force:

$$\langle F(\text{vac})\rangle = \frac{1}{6}\langle \delta\underline{r}\cdot\delta\underline{r}\rangle\nabla^2 F$$

$$= \frac{2}{3}\frac{\langle \delta\underline{r}\cdot\delta\underline{r}\rangle mMG}{r^4} \quad -(18)$$

If a negative spin connection vector is used in eq. (14) then:

$$\underline{F} = -\underline{\nabla}\phi - \underline{\omega}\phi \quad -(19)$$

and

$$\langle F(\text{vac}) \rangle = -\frac{2}{3} \frac{\langle \delta\underline{r} \cdot \delta\underline{r} \rangle}{r^4} mMG \quad -(20)$$

so:

$$F = -\frac{mMG}{r^2} - \frac{2}{3} mMG \frac{\langle \delta\underline{r} \cdot \delta\underline{r} \rangle}{r^4} \quad -(21)$$

The force from Einstein's theory of general relativity is eq. (3):

$$F = -\frac{mMG}{r^2} - \frac{3MGL^2}{mc^2 r^4} \quad -(22)$$

Therefore Einsteinian general relativity is a special case of the ECE2 force equation (19). EGR is defined by

$$\frac{2}{3} m \langle \delta\underline{r} \cdot \delta\underline{r} \rangle = \frac{3L^2}{mc^2} \quad -(23)$$

from a comparison of eqs. (21) and (22), i.e.

$$\langle \delta\underline{r} \cdot \delta\underline{r} \rangle = \frac{9L^2}{2m^2c^2} \quad -(24)$$

Finally use:

$$L^2 = \alpha m^2 MG \quad -(25)$$

where α is the half right latitude, so EGR is given by

$$\boxed{\langle \delta\underline{r} \cdot \delta\underline{r} \rangle = \frac{9}{2} \frac{\alpha MG}{c^2}} \quad -(26)$$

The so called "Schwarzschild radius"

$$r_0 = \frac{2MG}{c^2} \quad -(27)$$

so

$$\boxed{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle = \frac{9}{4} \alpha r_0} \quad - (28)$$

For a nearly circular orbit:

$$\alpha = r \quad - (29)$$

the radius of the orbit, so:

$$\boxed{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle = \frac{9}{4} r r_0} \quad - (29)$$

For the earth's orbit:

$$r = 1.495 \times 10^{10} \text{ m} \quad - (30)$$

and

$$r_0 = 3 \times 10^{5} \text{ m}$$

So the root mean square ~~or~~ vacuum fluctuation is about five orders of magnitude smaller than the radius of the orbit. From eqs. (12) and (26), the precession of the perihelion in the Einstein theory is given by: $- (31)$

$$\Delta \phi = \frac{6 \pi M G}{c^2 a (1 - \epsilon)} = \frac{6\pi}{a(1-\epsilon)} \cdot \frac{2}{9} \frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{\alpha}$$

i.e.

$$\boxed{\Delta \phi = \frac{4}{3} \pi \frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{a^2 (1 - \epsilon)(1 - \epsilon^2)}} \quad - (32)$$

The precession of the perihelion is due to the temporally averaged vacuum fluctuations.

The Einstein theory is a particular case of ECE2 field equation:

$$F = -\frac{mMG}{r^2} - \omega\Phi$$

$$= -\frac{mMG}{r^2} + \frac{mMG}{r}\omega \quad -(33)$$

nt of perihelion precession due to this force can be calculated with the original method as follows:

$$\frac{\partial F}{\partial r} = F' = \frac{2mMG}{r^3} - \frac{mMG}{r^2}\omega + \frac{mMG}{r}\frac{\partial \omega}{\partial r}$$

so

$$\frac{rF'}{F} = \frac{\frac{2mMG}{r^2} - \frac{mMG}{r}\omega + mMG\frac{\partial \omega}{\partial r}}{-\frac{mMG}{r^2} + \frac{mMG}{r}\omega}$$

$$= \frac{\frac{2}{r^2} + \frac{\omega}{r} - \frac{\partial \omega}{\partial r}}{-\frac{1}{r^2} + \frac{\omega}{r}} \quad -(34)$$

For small precessions, the spin connection is very small, so in the denominator:

$$-\frac{1}{r^2} + \frac{\omega}{r} \sim -\frac{1}{r^2} \quad -(35)$$

and

$$\frac{rF'}{F} \sim -2 - r^2\left(\frac{\omega}{r} - \frac{\partial \omega}{\partial r}\right) \quad -(36)$$

So

$$\phi = \pi\left(1 - r^2\left(\frac{\omega}{r} - \frac{\partial \omega}{\partial r}\right)\right)^{-1/2}$$

$$\sim \pi\left(1 + \frac{r^2}{2}\left(\frac{\omega}{r} - \frac{\partial \omega}{\partial r}\right)\right) \quad -(37)$$

Therefore the precession is:

$$\Delta\phi = \frac{r^2}{2}\left(\frac{\omega}{r} - \frac{\partial\omega}{\partial r}\right). \quad -(38)$$

At the perihelion:

$$r = a(1-\epsilon) \quad -(39)$$

From eq. (15):

$$\omega = \frac{2}{3}\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^3} \quad -(40)$$

So

$$\Delta\phi = \frac{1}{3}\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^2} - \frac{r^2}{2}\frac{\partial\omega}{\partial r} \quad -(41)$$

$$= \frac{1}{3}\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^2} - \frac{1}{3}\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{\omega r}\frac{\partial\omega}{\partial r}.$$

i.e.

$$\boxed{\Delta\phi = \frac{1}{3}\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^2}\left(1 - \frac{r}{\omega}\frac{\partial\omega}{\partial r}\right)} \quad -(42)$$

This precession is more general than the EGR precession and is deduced from the corresponding

Finally, from eq. (40):

$$\frac{\partial\omega}{\partial r} = \frac{2}{3}\frac{\partial}{\partial r}\left(\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^3}\right) \quad -(43)$$

If $\langle \delta \underline{r} \cdot \delta \underline{r} \rangle$ is a constant then

$$\frac{\partial\omega}{\partial r} = -2\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^4} \quad -(44)$$

$$\Delta\phi = \frac{1}{3}\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^2}\left(1 + 2\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{\omega r^3}\right)$$

8) From eq. (15):

$$\frac{2\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{\omega r^3} = 3 \quad -(46)$$

so

$$\boxed{\Delta \phi = \frac{4}{3} \frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^2}} \quad -(47)$$

The precession of the perihelion is:

$$\Delta \phi = \frac{4}{3} \frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{a^2(1-\epsilon)^2} \quad -(48)$$

from eq. (33).

A different sign of precession is obtained if the force equation is:

$$F = -\frac{mMG}{r^2} + \omega \phi \quad -(49)$$
