8(4): Checking the Note by Doug Lindstrom

The starting point is the tetrad postulate:

$$\Gamma^a_{\mu\nu} = \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu. \quad -(1)$$

By antisymmetry: $$\Gamma^a_{\mu\nu} = -\Gamma^a_{\nu\mu}. \quad -(2)$$

From Note 379(5):

$$\omega^a_{\mu b} q^b_\nu = \omega^a_\mu q_\nu \quad -(3)$$

For each a index, eq. (1) is therefore:

$$\Gamma_{\mu\nu} = \partial_\mu q_\nu + \omega_\mu q_\nu \quad -(4)$$

By definition,

$$\text{Trace } \Gamma_{\mu\nu} = \Gamma_{00} + \Gamma_{11} + \Gamma_{22} + \Gamma_{33} = 0 \quad -(5)$$

from eq. (2). Using the ECE postulate:

$$\partial_0 A_0 + \omega_0 A_0 + \sum_{i=1}^{3} \partial_i A_i + \omega_i A_i = 0 \quad -(6)$$

where:

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla} \right) \quad -(7)$$

$$A_\mu = \left(\frac{\phi}{c}, -\underline{A} \right) \quad -(8)$$

$$\omega_\mu = \left(\frac{\omega_0}{c}, -\underline{\omega} \right) \quad -(9)$$

So:

$$\frac{1}{c^2}\left(\frac{\partial \phi}{\partial t} + \omega_0 \phi \right) - \underline{\nabla} \cdot \underline{A} + \underline{\omega} \cdot \underline{A} = 0 \quad -(10)$$

The Lorenz condition is:

$$\frac{1}{c^2}\frac{\partial \phi}{\partial t} + \underline{\nabla} \cdot \underline{A} = 0 \quad -(11)$$

From eqs. (10) and (11):

$$\boxed{\frac{1}{c^2}\omega_0 \phi + \underline{\omega} \cdot \underline{A} = 2\underline{\nabla} \cdot \underline{A}} \quad -(12)$$

This is a new antisymmetry equation for any situation in electrodynamics.

The antisymmetry equations are therefore Eq. (12) and:

$$\underline{E} = -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{A}}{\partial t} - \omega_0 \underline{A} \quad -(13)$$

$$\frac{\partial A_Z}{\partial Y} + \frac{\partial A_Y}{\partial Z} = \omega_Y A_Z + \omega_Z A_Y \quad -(14)$$

$$\frac{\partial A_X}{\partial Z} + \frac{\partial A_Z}{\partial X} = \omega_Z A_X + \omega_X A_Z \quad -(15)$$

$$\frac{\partial A_Y}{\partial X} + \frac{\partial A_X}{\partial Y} = \omega_X A_Y + \omega_Y A_X \quad -(16)$$

Eq. (11) means:

$$\Box \phi = \rho / \epsilon_0 \quad -(17)$$

$$\Box \underline{A} = \mu_0 \underline{J} \quad -(18)$$

ϕ and $\underline{A}$ are found experimentally from the and $\underline{J}$ in the circuit.

Then $\underline{\omega}$ is found from eqs. (14) to (18). In order to find ω_0, eqs. (12) and (13) are solved simultaneously knowing ϕ, $\underline{A}$ and $\underline{\omega}$. Therefore A^{μ} and ω^{μ} can be found from the antisymmetry equations. The electric field strength $\underline{E}$ is found from eq. (13) and the magnetic flux density from:

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(19)$$

The electric field strength for circuit / vacuum interaction is:

$$\underline{E}_1 = \underline{\omega}\,\phi \quad -(20)$$

and the magnetic flux density for circuit / vacuum interaction is

$$\underline{B}_1 = -\underline{\omega} \times \underline{A} \quad -(21)$$
