381(6): Compatibility Check for Static Electric Field of Note 381(4).

The vector potential is given by Eq. (28):

$$\underline{A} = \frac{e_1 e_2}{4\pi\epsilon_0 \omega_0} \frac{\underline{r}}{r^3} \quad -(1)$$

where

$$\omega_0 = -\frac{c}{r} \quad -(2)$$

So:

$$\underline{A} = -\frac{e_1 e_2}{4\pi\epsilon_0 c r^2} \underline{r} \quad -(3)$$

$$= -\frac{e_1 e_2}{4\pi\epsilon_0 c r^2} \left(X \underline{i} + Y \underline{j} + Z \underline{k} \right)$$

where

$$r^2 = X^2 + Y^2 + Z^2 \quad -(4)$$

The vector spin connection is given by Eq. (21):

$$\underline{\omega} = \frac{\underline{r}}{r^2} = \frac{1}{r^2} \left(X \underline{i} + Y \underline{j} + Z \underline{k} \right) - (5)$$

The antisymmetry equations are:

$$\frac{\partial A_Z}{\partial Y} - \frac{\partial A_Y}{\partial Z} = \omega_Y A_Z - \omega_Z A_Y \quad -(6)$$

$$\frac{\partial A_X}{\partial Z} - \frac{\partial A_Z}{\partial X} = \omega_Z A_X - \omega_X A_Z \quad -(7)$$

$$\frac{\partial A_Y}{\partial X} - \frac{\partial A_X}{\partial Y} = \omega_X A_Y - \omega_Y A_X \quad -(8)$$

2) From eq. (3):

$$\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = 0 \quad -(9)$$

$$\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = 0 \quad -(10)$$

$$\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = 0 \quad -(11)$$

From eqs. (3) and (5):

$$\omega_y A_z - \omega_z A_y = 0 \quad -(12)$$

$$\omega_z A_x - \omega_x A_z = 0 \quad -(13)$$

$$\omega_x A_y - \omega_y A_x = 0 \quad -(14)$$

So the antisymmetry laws are obeyed, Q.E.D.