

COMPLETE SOLUTION OF THE ECE2 FIELD EQUATIONS: THE BIEFELD BROWN  
COUNTER GRAVITATIONAL EFFECT.

by

M. W. Evans and H. Eckardt

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ABSTRACT

The homogeneous field equations of ECE2 field theory are used together with the antisymmetry laws, to solve for the spin connection four vector and vector potential three vector. Example solutions are given using hand calculations checked with computer algebra. It is possible to obtain the general solutions computationally using a package such as Mathematica. Some solutions can be obtained with Maxima. A straightforward explanation is given for the Biefeld Brown counter gravitational effect.

Keywords: ECE2 field equations, solutions for the spin connection and vector potential.

UFT 380

## 1. INTRODUCTION

In recent papers of this series {1 - 12}, the important discovery has been made that the ECE lagrangian produces both forward and retrograde precession in planar orbits, thus going beyond the Einsteinian general relativity (EGR). This work suggests that general solutions of the ECE2 field equations are needed, both for gravitation and electromagnetism. In this paper the homogeneous field equations of ECE2 gravitation and electromagnetism are solved together with the antisymmetry laws {1 - 12}. Some example solutions are described, and the general problem can be solved with a package such as Mathematica. The Maxima package can solve certain types of problem.

This paper is a brief synopsis of detailed calculations found in the notes accompanying UFT380 on [www.aias.us](http://www.aias.us) and [www.upitec.org](http://www.upitec.org). These notes are an intrinsic part of the paper. Note 380(1) gives a method of evaluating the vector potential three vector for gravitation (Q) and electromagnetism (A) and the spin connection four vector, ( $\omega^\mu$ ) which is an intrinsic part of geometry and which is the same for gravitation and electromagnetism. The note defines the field tensor, the antisymmetry laws and the Gauss law for gravitation and electromagnetism. Note 380(2) combines electrodynamics and gravitation to give a straightforward explanation of the Biefeld Brown counter gravitational effect. Note 380(3) describes schemes of calculation for  $\omega^\mu$  and Q. Note 380(4) gives Cartesian equations for the vector potential components and the components of the spin connection four vector. Note 350(5) introduces the Faraday law of induction for gravitation and electromagnetism and shows that the problem is exactly determined, seven equations in seven unknowns. This note is used as the basis for Section 2. Note 380(6) gives a hand calculated plane wave solution of the problem.

In Section 3 some example solutions and schemes of calculation and computation are solved with Maxima and graphed en route to the general solution that can be used throughout the physical sciences and engineering.

## 2. THE SEVEN EQUATIONS IN SEVEN UNKNOWNNS

Consider the homogeneous field equations and antisymmetry laws of ECE2 gravitation. The homogeneous field equations are the Gauss law:

$$\underline{\nabla} \cdot \underline{\Omega} = 0 \quad - (1)$$

and the Faraday law of induction:

$$\underline{\nabla} \times \underline{g} + \frac{\partial \underline{\Omega}}{\partial t} = \underline{0} \quad - (2)$$

Here  $\underline{g}$  is the gravitational field and  $\underline{\Omega}$  the gravitomagnetic field, respectively defined by:

$$\underline{g} = -\frac{\partial \underline{Q}}{\partial t} - \underline{\omega}_0 \underline{Q} = -\underline{\nabla} \Phi + \underline{\omega} \Phi \quad - (3)$$

and

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q} - \underline{\omega} \times \underline{Q} \quad - (4)$$

Here  $\underline{Q}$  is the vector potential of gravitation,  $\Phi$  is the scalar potential,  $\underline{\omega}$  is the spin connection vector. and  $\underline{\omega}_0$  is the timelike part of the spin connection four vector.

In terms of Cartesian components, Eq. (1) gives

$$\begin{aligned} & Q_x \left( \frac{\partial \omega_z}{\partial y} - \frac{\partial \omega_y}{\partial z} \right) + Q_y \left( \frac{\partial \omega_x}{\partial z} - \frac{\partial \omega_z}{\partial x} \right) + Q_z \left( \frac{\partial \omega_y}{\partial x} - \frac{\partial \omega_x}{\partial y} \right) - (5) \\ & = \omega_x \left( \frac{\partial Q_z}{\partial y} - \frac{\partial Q_y}{\partial z} \right) + \omega_y \left( \frac{\partial Q_x}{\partial z} - \frac{\partial Q_z}{\partial x} \right) + \omega_z \left( \frac{\partial Q_y}{\partial x} - \frac{\partial Q_x}{\partial y} \right) \end{aligned}$$

as described in the Notes. Eqs. (2) to (4) give:

$$\frac{d}{dt} (\underline{\omega} \times \underline{Q}) + \nabla \times (\omega_0 \underline{Q}) = 0 \quad - (6)$$

The antisymmetry laws from Eq. ( 4 ) are derived in Note 380(4), and are:

$$\left( \frac{\partial}{\partial y} - \omega_1 \right) Q_z = - \left( \frac{\partial}{\partial z} - \omega_2 \right) Q_y \quad - (7)$$

$$\left( \frac{\partial}{\partial z} - \omega_2 \right) Q_x = - \left( \frac{\partial}{\partial x} - \omega_x \right) Q_z \quad - (8)$$

$$\left( \frac{\partial}{\partial x} - \omega_x \right) Q_y = - \left( \frac{\partial}{\partial y} - \omega_1 \right) Q_x \quad - (9)$$

These can be expressed as:

$$\frac{\partial Q_z}{\partial y} + \frac{\partial Q_y}{\partial z} = \omega_2 Q_y + \omega_1 Q_z \quad - (10)$$

$$\frac{\partial Q_x}{\partial z} + \frac{\partial Q_z}{\partial x} = \omega_x Q_z + \omega_2 Q_x \quad - (11)$$

$$\frac{\partial Q_y}{\partial x} + \frac{\partial Q_x}{\partial y} = \omega_x Q_y + \omega_1 Q_x \quad - (12)$$

Eq. ( 6 ) gives three further equations:

$$\frac{d}{dt} (\omega_1 Q_z - \omega_2 Q_y) + \omega_0 \left( \frac{\partial Q_z}{\partial y} - \frac{\partial Q_y}{\partial z} \right) + Q_z \frac{\partial \omega_0}{\partial y} - Q_y \frac{\partial \omega_0}{\partial z} = 0 \quad - (13)$$

$$\frac{d}{dt} (\omega_2 Q_x - \omega_x Q_z) + \omega_0 \left( \frac{\partial Q_x}{\partial z} - \frac{\partial Q_z}{\partial x} \right) + Q_x \frac{\partial \omega_0}{\partial z} - Q_z \frac{\partial \omega_0}{\partial x} = 0 \quad - (14)$$

$$\frac{d}{dt} (\omega_x Q_y - \omega_1 Q_x) + \omega_0 \left( \frac{\partial Q_y}{\partial x} - \frac{\partial Q_x}{\partial y} \right) + Q_y \frac{\partial \omega_0}{\partial x} - Q_x \frac{\partial \omega_0}{\partial y} = 0 \quad - (15)$$

so the problem is exactly determined, to solve Eqs. ( 5 ), ( 10 ) to ( 12 ) and

( 13 ) to ( 15 ) for the three components of  $\underline{Q}$  and the four components of  $\underline{\omega}^M$ .

In general this system can be solved by computer using a package such as Mathematica, and the package Maxima can solve some problems. The general solution can address many situations in the physical sciences and engineering.

Consider the approximation:

$$\underline{\Omega} \ll \underline{g} \approx \underline{0} \quad - (16)$$

which means that the gravitomagnetic field is much smaller than the gravitational field. The approximation ( 16 ) means that:

$$\underline{\nabla} \times \underline{Q} = \underline{\omega} \times \underline{Q} \quad - (17)$$

and as shown in Note 380(5), the antisymmetry laws ( 10 ) to ( 12 ) and Eq. ( 17 ) lead to:

$$\frac{\partial Q_x}{\partial Z} = \omega_z Q_x \quad - (18)$$

and:

$$\frac{\partial Q_y}{\partial Z} = \omega_z Q_y \quad - (19)$$

so:

$$Q_x = Q_y. \quad - (20)$$

A solution of Eqs. ( 18 ) and ( 20 ) is the wave:

$$Q_x = Q_y = i Q_0 \exp(i(\omega t - \kappa_z Z)) \quad - (21)$$

where  $\omega$  is its angular frequency at instant  $t$ , and  $\kappa_z$  is its wave vector at position  $Z$ .

The use of Maxima confirms that the general solution has the format of Eq. ( 21 ). From

Eq. (21):

$$\kappa_z = \omega_z \quad - (22)$$

so for a unidirectional wave in the Z axis, as shown in Note 380(5):

$$Q_x = Q_y = Q_z = iQ_0 \exp(i(\omega t - \kappa_z z)) \quad - (23)$$

If it is further assumed that:

$$\frac{\partial \omega_0}{\partial x} = \frac{\partial \omega_0}{\partial t} = \frac{\partial \omega_0}{\partial z} = 0 \quad - (24)$$

then the solution of the seven relevant equations described already is:

$$Q_x = Q_y = Q_z = iQ_0 \exp(i(\omega t - \kappa_z z)) \quad - (25)$$

and

$$\omega^\mu = \left( \frac{\omega}{c}, \frac{\kappa}{-} \right) \quad - (26)$$

with:

$$\underline{\Omega} \sim \underline{0} \quad - (27)$$

Therefore the spin connection is the wave four vector:

$$\omega^\mu = \kappa^\mu \quad - (28)$$

The system quantizes to:

$$p^\mu = \hbar \kappa^\mu = \hbar \omega^\mu \quad - (29)$$

where  $p^\mu$  the energy momentum four vector:

$$P^{\mu} = \left( \frac{E}{c}, \underline{P} \right) \quad - (30)$$

The energy momentum of a graviton is the spin connection four vector within the reduced Planck constant  $\hbar$ .

For ECE2 electrodynamics the corresponding solution for the vector potential is:

$$A_x = A_y = A_z = i A^{(0)} \exp(i(\omega t - k_z z)) \quad - (31)$$

and the spin connection four vector is the same. Further details are given in Note 380(5).

Another possible solution is given in Note 380(6) by evaluating the spin connection four vector from the vector potential plane wave {1 - 12}:

$$\underline{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) \exp(-i(\omega t - k_z z)) \quad - (32)$$

for which the antisymmetry laws reduce to:

$$\frac{\partial A_x^{(1)}}{\partial z} = \omega_z A_x^{(1)} \quad - (33)$$

$$\frac{\partial A_y^{(1)}}{\partial z} = \omega_z A_y^{(1)} \quad - (34)$$

$$-\omega_x A_y^{(1)} = \omega_y A_x^{(1)} \quad - (35)$$

from which:

$$\omega_z = 0 \quad - (36)$$

and

$$i\omega_x = \omega_y \quad - (37)$$

as shown in Note 380(6). Eqs. ( 36 ) and ( 37 ) are satisfied by the complex conjugate spin connection plane wave:

$$\underline{\omega}^{(2)} = \frac{\omega^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) \exp(i(\omega t - \kappa z)) \quad (38)$$

It follows as in Note 380(6) that

$$\omega_0 = 0. \quad (39)$$

Therefore if it is assumed that:

$$\underline{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) \exp(-i(\omega t - \kappa z)) \quad (40)$$

then a possible solution is

$$\underline{\omega}^{(2)} = \frac{\omega^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) \exp(i(\omega t - \kappa z)) \quad (41)$$

so if the vector potential is assumed to be a plane wave, the spin connection is spacelike and also a plane wave. This is a plane wave of spacetime or the aether. In Section 3 other solutions are given using Maxima and graphed.

Finally, the Biefeld Brown counter gravitation effect {1 - 12} can be given a simple explanation using the inhomogeneous Coulomb laws of electromagnetism and gravitation, respectively:

$$-\nabla^2 \phi + \underline{\nabla} \cdot (\phi \underline{\omega}) = \frac{\rho}{\epsilon_0} \quad (42)$$

and

$$-\nabla^2 \Phi + \underline{\nabla} \cdot (\Phi \underline{\omega}) = 4\pi G \rho / m \quad (43)$$

as in Note 380(2). The spin connection vector  $\underline{\omega}$  is the same in both equations. Here  $\phi$  is the electromagnetic scalar potential,  $\Phi$  is the gravitational scalar potential.

$\rho$  is the electric charge density,  $\rho_m$  is the mass density,  $\epsilon_0$  is the vacuum permittivity and  $G$  is Newton's constant. For a particle with charge  $e$  and mass  $m$  the total potential energy in joules is:

$$U = e\phi + m\Phi \quad - (44)$$

so:

$$-\nabla^2 U + \nabla \cdot (\underline{U} \underline{\omega}) = \frac{e\rho}{\epsilon_0} + 4\pi m G \frac{\rho}{m} \quad - (45)$$

and the total force on the particle is:

$$\underline{F} = -\nabla U \quad - (46)$$

By adding Eqs. ( 44 ) and ( 45 ) it is found that there are cross effects such as:

$$-\nabla^2 \Phi + \nabla \cdot (\underline{\Phi} \underline{\omega}) = \frac{e}{m} \frac{\rho}{\epsilon_0} + \dots \quad - (47)$$

where an electric charge density (as in a Biefeld Brown capacitor) can influence the gravitational potential and cause counter gravitation, Q. E. D..

This effect is developed and graphed in Section 3.

### 3. COMPUTATION AND GRAPHICS

By Dr. Horst Eckardt

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