380(1): Evaluation of the $\underline{Q}$ and Spin Connection Vectors

From Note 379 it was found that the antisymmetry condition reduces the following four equations in the limit:

$$\Phi = \frac{mG}{r} \quad -(1)$$

The first two are force equations:

$$\ddot{X} = -\frac{mG}{(X^2+Y^2)^{1/2}}\left(\frac{X}{X^2+Y^2} - \omega_X\right) \quad -(2)$$

$$\ddot{Y} = -\frac{mG}{(X^2+Y^2)^{1/2}}\left(\frac{Y}{X^2+Y^2} - \omega_Y\right), \quad -(3)$$

where the spin connection vector is:

$$\underline{\omega} = \omega_X \underline{i} + \omega_Y \underline{j} \quad -(4)$$

By antisymmetry:

$$\left(\frac{d}{dt} + \omega_0\right) Q_X = \frac{mG}{(X^2+Y^2)^{1/2}}\left(\frac{X}{X^2+Y^2} - \omega_X\right) - (5)$$

$$\left(\frac{d}{dt} + \omega_0\right) Q_Y = \frac{mG}{(X^2+Y^2)^{1/2}}\left(\frac{Y}{X^2+Y^2} - \omega_Y\right) - (6)$$

There are five unknowns, ω_0, ω_X, ω_Y, Q_X and Q_Y, and to determine these uniquely requires more information. The gravitomagnetic field equation is:

$$\underline{\nabla} \cdot \underline{\Omega} = 0 \quad -(7)$$

the assumed absence of a gravitomagnetic monopole.

2) The gravitomagnetic field is found from:

$$\sigma_{12} = (\partial_1 + \omega_1) Q_2 - (\partial_2 + \omega_2) Q_1 \quad -(8)$$
$$\sigma_{13} = (\partial_1 + \omega_1) Q_3 - (\partial_3 + \omega_3) Q_1 \quad -(9)$$
$$\sigma_{23} = (\partial_2 + \omega_2) Q_3 - (\partial_3 + \omega_3) Q_2 \quad -(10)$$

where the field tensor is:

$$\sigma_{\mu\nu} = \begin{bmatrix} 0 & -g_1/c & -g_2/c & -g_3/c \\ g_1/c & 0 & \Omega_3 & -\Omega_2 \\ g_2/c & -\Omega_3 & 0 & \Omega_1 \\ g_3/c & \Omega_2 & -\Omega_1 & 0 \end{bmatrix} \quad -(11)$$

Therefore

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q} - \underline{\omega} \times \underline{Q} \quad -(12)$$

From eqs. (7) and (12)

$$\underline{\nabla} \cdot \underline{\omega} \times \underline{Q} = \underline{Q} \cdot \underline{\nabla} \times \underline{\omega} - \underline{\omega} \cdot \underline{\nabla} \times \underline{Q} = 0 \quad -(13)$$

The antisymmetry law from eqs. (8) to (10) is:

$$(\partial_1 + \omega_1) Q_2 = -(\partial_2 + \omega_2) Q_1 \quad -(14)$$
$$(\partial_1 + \omega_1) Q_3 = -(\partial_3 + \omega_3) Q_1 \quad -(15)$$
$$(\partial_2 + \omega_2) Q_3 = -(\partial_3 + \omega_3) Q_2 \quad (16)$$

For planar orbital motion:

$$(\partial_1 + \omega_1) Q_2 = -(\partial_2 + \omega_2) Q_1 \quad -(17)$$

3) so
$$\left(\frac{\partial}{\partial X} - \omega_X\right) Q_Y = -\left(\frac{\partial}{\partial Y} - \omega_Y\right) Q_X \quad -(18)$$

for planar motion in general.

Eqs. (2), (3), (5), (6) and (18) are five equations in five unknowns, ω_0, ω_X, ω_Y, Q_X and Q_Y. This set of equations can be solved numerically.

In addition, the orbit from eqns. (2) and (3) must be the same as the orbit from the hamiltonian:
$$H = \gamma m c^2 + U \quad -(19)$$

and the lagrangian:
$$\mathcal{L} = -\frac{mc^2}{\gamma} - U \quad -(20)$$

The solution for $\underline{\omega}$ and $\underline{Q}$ must obey the equation:
$$\underline{\nabla} \cdot \underline{\omega} \times \underline{Q} = \underline{0} \quad -(21)$$

Eq. (21) is obeyed by planar motion in which:
$$\underline{\omega} = \omega_X \underline{i} + \omega_Y \underline{j} \quad -(22)$$
$$\underline{Q} = Q_X \underline{i} + Q_Y \underline{j} \quad -(23)$$

if
$$\frac{\partial}{\partial Z}\left(\omega_X Q_Y - \omega_Y Q_X\right) = 0 \quad -(24)$$

This is true automatically if $\underline{\omega}$ and $\underline{Q}$ are

4) defined is a plane XY and have no Z dependence.

In general the ECE2 gravitational field equations imply that $\underline{\Omega}$ exists if $\underline{g}$ exists. The components of the gravitomagnetic field are in general:

$$\Omega_1 = (\partial_2 + \omega_2) Q_3 - (\partial_3 + \omega_3) Q_2 \quad -(18)$$

$$\Omega_2 = (\partial_3 + \omega_3) Q_1 - (\partial_1 + \omega_1) Q_3 \quad -(19)$$

$$\Omega_3 = (\partial_1 + \omega_1) Q_2 - (\partial_2 + \omega_2) Q_1 \quad -(20)$$

For a planar orbit:

$$\Omega_3 = (\partial_1 + \omega_1) Q_2 - (\partial_2 + \omega_2) Q_1 \quad -(21)$$

i.e.

$$\Omega_Z = \left(\frac{\partial}{\partial X} - \omega_X \right) Q_Y - \left(\frac{\partial}{\partial Y} - \omega_Y \right) Q_X \quad -(22)$$

so the gravitomagnetic field is perpendicular to the plane of the orbit. From eqs. (18) and (22):

$$\Omega_Z = 2 \left(\frac{\partial}{\partial X} - \omega_X \right) Q_Y \quad -(23)$$

$$= -2 \left(\frac{\partial}{\partial Y} - \omega_Y \right) Q_X$$

Newtonian Limit

In this limit:

$$\omega_0 = 0 \quad -(24)$$

and

$$\omega_X = \omega_Y = 0 \quad -(25)$$

so:

5)

$$\ddot{X} = -\frac{\partial Q_X}{\partial t} = \frac{mG\,X}{(X^2+Y^2)^{3/2}} \quad -(26)$$

and

$$\ddot{Y} = -\frac{\partial Q_Y}{\partial t} = \frac{mG\,Y}{(X^2+Y^2)^{3/2}} \quad -(27)$$

Eq. (18) reduces to:

$$\frac{\partial Q_Y}{\partial X} = -\frac{\partial Q_X}{\partial Y} \quad -(28)$$

and

$$\Omega_Z = 2\frac{\partial Q_Y}{\partial X} = -2\frac{\partial Q_X}{\partial Y} \quad -(29)$$

$$= 0$$

because in Newtonian gravitation there is no gravito-magnetic field. The orbit from eqs. (26) and (27) is an ellipse. In the presence of a spin connection the orbit is a forward or retrograde precessing ellipse.

Zero Gravitation Limit

In this case:

$$\ddot{X} = \ddot{Y} = 0 \quad -(30)$$

so:

$$\omega_X = \frac{X}{X^2+Y^2} \quad -(31)$$

$$\omega_Y = \frac{Y}{X^2+Y^2} \quad -(32)$$

Therefore eqs. (5) and (6) reduce to:

$$\left(\frac{\partial}{\partial t} + \omega_0\right) Q_X = \left(\frac{\partial}{\partial t} + \omega_0\right) Q_Y = 0 \quad -(33)$$

6) Eq. (18) reduces to:

$$\left(\frac{\partial}{\partial X} - \frac{X}{X^2+Y^2}\right) Q_Y = -\left(\frac{\partial}{\partial Y} - \frac{Y}{X^2+Y^2}\right) Q_X \quad -(34)$$

From eq. (33): $Q_X = Q_Y \quad -(35)$

So there are two equations in two unknowns:

$$\left(\frac{\partial}{\partial t} + \omega_0\right) Q_X = 0 \quad -(36)$$

and

$$\left(\frac{\partial}{\partial X} - \frac{X}{X^2+Y^2}\right) Q_X = -\left(\frac{\partial}{\partial Y} - \frac{Y}{X^2+Y^2}\right) Q_X \quad -(37)$$

So any reports of zero gravitation can be analyzed in this way. Counter gravitation means changing the signs of $\ddot{X}$ and $\ddot{Y}$ in eqs. (2) and (3).

The above analysis depends on the assumption (1), i.e. that the scalar potential is essentially Newtonian, and that orbital precessions are small. More generally

$$\underline{g} = -\underline{\nabla}\Phi + \Phi\underline{\omega} = -\frac{\partial \underline{Q}}{\partial t} - \omega_0 \underline{Q} \quad -(38)$$

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m \quad -(39)$$

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q} - \underline{\omega} \times \underline{Q} \quad -(40)$$

$$\underline{\nabla} \cdot \underline{\Omega} = 0 \quad -(41)$$