77(5): New Constraint Equation

The results of the computer algebra are:

$$\ddot{X} = \frac{mG}{\gamma^2(X^2+Y^2)^{3/2}}\left(\frac{\dot{X}\dot{Y}Y + X\dot{X}^2}{c^2} - X\right) \quad -(1)$$

$$= -\frac{mG\,X}{\gamma^3(X^2+Y^2)^{3/2}}$$

and

$$\ddot{Y} = \frac{mG}{\gamma^2(X^2+Y^2)^{3/2}}\left(\frac{\dot{Y}\dot{X}X + Y\dot{Y}^2}{c^2} - Y\right) \quad -(2)$$

$$= -\frac{mG\,Y}{\gamma^3(X^2+Y^2)^{3/2}}$$

Therefore:

$$\left(1 - \frac{1}{\gamma}\right)X = \frac{\dot{X}\dot{Y}Y + X\dot{X}^2}{c^2} \quad -(3)$$

and

$$\left(1 - \frac{1}{\gamma}\right)Y = \frac{\dot{X}\dot{Y}X + Y\dot{Y}^2}{c^2} \quad -(4)$$

Therefore:

$$\dot{X}^2 + \frac{Y\dot{Y}\dot{X}}{X} = \dot{Y}^2 + \frac{X\dot{X}\dot{Y}}{Y} \quad -(5)$$

so

$$\boxed{\dot{X}\left(\dot{X} + \frac{Y}{X}\dot{Y}\right) = \dot{Y}\left(\dot{Y} + \frac{X}{Y}\dot{X}\right)} \quad -(6)$$

Therefore:

$$\dot{X}^2 - \dot{Y}^2 + \frac{Y}{X}\dot{X}\dot{Y} - \frac{X}{Y}\dot{X}\dot{Y} = 0 \quad -(7)$$

i.e.

$$\dot{X}^2 - \dot{Y}^2 = \frac{\dot{X}\dot{Y}}{XY}\left(X^2 - Y^2\right) \quad -(8)$$

Therefore eq. (8) develops eq. (1) into a self-consistent equation, and similarly for eq. (2). Eq. (8) can be written as:

$$\frac{\dot{X}^2 - \dot{Y}^2}{\dot{X}\dot{Y}} = \frac{X^2 - Y^2}{XY} \quad -(9)$$

i.e.

$$\boxed{\frac{\dot{X}}{\dot{Y}} - \frac{\dot{Y}}{\dot{X}} = \frac{X}{Y} - \frac{Y}{X}} \quad -(10)$$

From the Coulomb law:

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_{\underline{m}} \quad -(11)$$

of ECE2 gravitational field theory:

$$X\kappa_X + Y\kappa_Y = -1 \quad -(12)$$

For assumed constant κ_X and κ_Y:

$$\kappa_X \dot{X} + \kappa_Y \dot{Y} = 0 \quad -(13)$$

Eq. (13) assumes that κ_X and κ_Y are independent of time. From eq. (13):

$$\frac{\dot{X}}{\dot{Y}} = -\frac{\kappa_Y}{\kappa_X} \quad -(14)$$

) So:

$$\frac{\kappa_x}{\kappa_y} - \frac{\kappa_y}{\kappa_x} = \frac{X}{Y} - \frac{Y}{X} \quad (15)$$

Fm eq. (12):

$$\frac{X}{Y} = -\frac{1}{\kappa_x}\left(\frac{1}{Y} + \kappa_y\right) \quad (16)$$

so

$$\frac{Y}{X} = -\frac{\kappa_x}{\left(\frac{1}{Y} + \kappa_y\right)} \quad (17)$$

Therefore

$$\frac{\kappa_x}{\left(\frac{1}{Y} + \kappa_y\right)} - \frac{1}{\kappa_x}\left(\frac{1}{Y} + \kappa_y\right)$$

$$= \frac{\kappa_x}{\kappa_y} - \frac{\kappa_y}{\kappa_x} \quad (18)$$

Therefore Y can be found in terms of κ_x and κ_y. Similarly X can be found in terms of κ_x and κ_y.

Retrograde Precessions

These follow from:

$$\gamma^3 \ddot{\underline{r}} = -mG\frac{\underline{r}}{r^3} \quad (19)$$

i.e.

$$\ddot{X} = -\frac{mGX}{\gamma^3(X^2+Y^2)^{3/2}} \quad (20)$$

$$\ddot{Y} = -\frac{mGY}{\gamma^3(X^2+Y^2)^{3/2}} \quad (21)$$

...rward Precessions

These follow from:

4)
$$\mathcal{L} = -mc^2\left(1-\frac{\dot{x}^2+\dot{y}^2}{c^2}\right)^{1/2} + \frac{mMG}{(x^2+y^2)^{1/2}} \quad -(22)$$

and
$$\frac{\partial \mathcal{L}}{\partial x} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} \quad -(23)$$
$$\frac{\partial \mathcal{L}}{\partial y} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{y}} \quad -(24)$$

giving:
$$\ddot{x} = \frac{MG}{\gamma^2(x^2+y^2)^{3/2}}\left(\frac{\dot{x}\dot{y}y+x\dot{x}^2}{c^2}-x\right) \quad -(25)$$
and
$$\ddot{y} = \frac{MG}{\gamma^2(x^2+y^2)^{3/2}}\left(\frac{\dot{y}\dot{x}x+y\dot{y}^2}{c^2}-y\right) \quad -(26)$$

The two types of precession are related by eq. (10):
$$\frac{\dot{x}}{\dot{y}} - \frac{\dot{y}}{\dot{x}} = \frac{x}{y} - \frac{y}{x} \quad -(27)$$

and by the Coulomb law (11). Therefore a retrograde precession is transformed into a forward precession by starting with eqs. (20) and (21), then applying eq. (27). A forward precession is transformed into a retrograde precession by starting with eqs. (25) and (26) and applying eq. (27).