377(3): Checking Calculations

We have:

$$\frac{\partial g_x}{\partial x} = -\frac{mG}{r^3}\frac{d}{dx}\left(\frac{x}{(x^2+y^2)^{3/2}}\right)$$

$$= -\frac{mG}{r^3}\left(\frac{(x^2+y^2)^{3/2} - 3x^2(x^2+y^2)^{1/2}}{(x^2+y^2)^3}\right) \quad -(1)$$

$$= -\frac{mG}{r^3}\left(\frac{1}{(x^2+y^2)^{3/2}} - \frac{3x^2}{(x^2+y^2)^{5/2}}\right)$$

Similarly: $\frac{\partial g_y}{\partial y} = -\frac{mG}{r^3}\left(\frac{1}{(x^2+y^2)^{3/2}} - \frac{3y^2}{(x^2+y^2)^{5/2}}\right) \quad -(2)$

So $\frac{\partial g_x}{\partial x} + \frac{\partial g_y}{\partial y} = \frac{mG}{r^3}\left(\frac{3(x^2+y^2)}{(x^2+y^2)^{5/2}} - \frac{2}{(x^2+y^2)^{3/2}}\right)$

$$= \frac{mG}{r^3(x^2+y^2)^{3/2}} \quad \checkmark \quad -(3)$$

$$= \kappa_x g_x + \kappa_y g_y = -\frac{mG}{r^3(x^2+y^2)^{3/2}}(\kappa_x x + \kappa_y y)$$

So $x\kappa_x + y\kappa_y = -1 \quad -(4)$

$$x = -\frac{1}{\kappa_x}(1 + \kappa_y y) \quad -(5) \checkmark$$

$$y = -\frac{1}{\kappa_y}(1 + \kappa_x x) \quad -(6) \checkmark$$

$$\rho_m = \frac{M}{4\pi r^3(x^2+y^2)^{3/2}} \quad -(7) \checkmark$$