377(2): Orbital Precession in Terms of the ECE2 Vector $\underline{\kappa}$.

The relevant equations are:

$$\underline{F} = \gamma^3 m \underline{g} = -mMG \frac{\underline{r}}{r^3} \quad -(1)$$

and

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m \quad -(2)$$

where

$$\gamma^3 = \left(1 - \frac{\dot{x}^2 + \dot{y}^2}{c^2}\right)^{-3/2} \quad -(3)$$

and

$$\underline{g} = \ddot{\underline{r}}. \quad -(4)$$

So:

$$\gamma^3 g_x = -MG \frac{X}{(X^2+Y^2)^{3/2}} \quad -(5)$$

$$\gamma^3 g_Y = -MG \frac{Y}{(X^2+Y^2)^{3/2}} \quad -(6)$$

and

$$\frac{\partial g_X}{\partial X} + \frac{\partial g_Y}{\partial Y} = \kappa_X g_X + \kappa_Y g_Y = 4\pi G \rho_m \quad -(7)$$

Define:

$$g_X = -\frac{MG}{\gamma^3} \frac{X}{(X^2+Y^2)^{3/2}} \quad -(8)$$

$$g_Y = -\frac{MG}{\gamma^3} \frac{Y}{(X^2+Y^2)^{3/2}} \quad -(9)$$

The γ^3 factor has no dependence on X and Y so:

$$\frac{\partial g_x}{\partial X} = -\frac{mG}{\gamma^3}\frac{\partial}{\partial X}\left(\frac{X}{(X^2+Y^2)^{3/2}}\right) \quad -(10)$$

and

$$\frac{\partial g_Y}{\partial Y} = -\frac{mG}{\gamma^3}\frac{\partial}{\partial Y}\left(\frac{Y}{(X^2+Y^2)^{3/2}}\right) \quad -(11)$$

So:

$$\frac{\partial g_X}{\partial X} + \frac{\partial g_Y}{\partial Y} = \frac{2mG}{\gamma^3(X^2+Y^2)^{3/2}} \quad -(12)$$

$$= \kappa_X g_X + \kappa_Y g_Y$$

From eqs. (5), (6) and (12):

$$\boxed{\kappa_X X + \kappa_Y Y = -2} \quad -(13)$$

It also follows that:

$$\boxed{\rho_m = \frac{m}{2\pi\gamma^3(X^2+Y^2)^{3/2}}} \quad -(14)$$

The solution of Eq. (13) is:

$$X = -\frac{1}{\kappa_X}(2+\kappa_Y Y) \quad -(15)$$

$$Y = -\frac{1}{\kappa_Y}(2+\kappa_X X) \quad -(16)$$

in general.

So the orbit is determined by:

$$\ddot{X} = -mG\left(1 - \frac{\dot{x}^2 + \dot{y}^2}{c^2}\right)^{3/2} \frac{X}{(x^2+y^2)^{3/2}} \quad -(17)$$

$$\ddot{Y} = -mG\left(1 - \frac{\dot{x}^2 + \dot{y}^2}{c^2}\right)^{3/2} \frac{Y}{(x^2+y^2)^{3/2}} \quad -(18)$$

$$X = -\frac{1}{\kappa_x}\left(2 + \kappa_y Y\right) \quad -(19)$$

$$Y = -\frac{1}{\kappa_y}\left(2 + \kappa_x X\right) \quad -(20)$$

In general, eqs. (17) to (20) must be solved simultaneously, using κ_x and κ_y as input parameters.

One particular solution is:

$$\kappa_x = -\frac{1}{X}, \quad -(21)$$

$$\kappa_y = -\frac{1}{Y}. \quad -(22)$$

and the set of eqs. (17) to (20) reduces to eqs. (17) and (18).

The nature of the precession is determined by κ_x and κ_y.