375(6): Definition of α and ϵ of the Ellipse

If a is the semi major and b the semi minor axis then

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad -(1)$$

where

$$x = c + r\cos\phi \quad -(2)$$

$$y = r\sin\phi. \quad -(3)$$

The eccentricity is:

$$\epsilon = \left(1 - \frac{b^2}{a^2}\right)^{1/2} = \frac{c}{a} \quad -(4)$$

The half right latitude is:

$$\alpha = a\left(1-\epsilon^2\right) = \frac{b^2}{a} \quad -(5)$$

From eqs. (2) and (3):

$$r^2 = (x-c)^2 + y^2 \quad -(6)$$

We have

$$a = \frac{\alpha}{1-\epsilon^2}, \quad b = \frac{\alpha}{(1-\epsilon^2)^{1/2}} \quad -(7)$$

Therefore α and ϵ can be found by measurements of the semi major axis a and semi minor axis b.

From eqs. (1) and (7):

$$x^2(1-\epsilon^2)^2 + y^2(1-\epsilon^2) = \alpha^2. \quad -(8)$$

We have

$$\ddot{x} = -\frac{mG}{(x^2+y^2)^{3/2}}\, x \quad -(9)$$

$$\ddot{y} = -\frac{mG}{(x^2+y^2)^{3/2}}\, y \quad -(10)$$

2) If eqns. (9) and (10) are solved the results for X and Y can be used to test the validity of eq. (8), given the experimental α and ϵ. In the relativistic case we must solve:

$$\ddot{\underline{r}} = \frac{\gamma m G}{r^3}\left(\frac{\underline{v}(\underline{v}\cdot\underline{r})}{c^2} - \underline{r}\right) \quad (11)$$

for X and Y. Then Eq (8) will no longer be true because of relativistic effects.