

375 (5): Lagrangian for Hulse Taylor Pulsar and Falvo of the
Einstein Theory.



With reference to Fig (1), the pulsar of mass m_p and companion star of mass m_c orbit the centre of mass CM. Define:

$$\underline{r} = \underline{r}_1 - \underline{r}_2 \quad (1)$$

and the separation between m_p and m_c is:

$$r = |\underline{r}| \quad (2)$$

The Lagrangian (Meria and Thornton chapter 7) is:

$$L = \frac{1}{2} m_p \dot{\underline{r}}_1 \cdot \dot{\underline{r}}_1 + \frac{1}{2} m_c \dot{\underline{r}}_2 \cdot \dot{\underline{r}}_2 - U(r) \quad (3)$$

where the gravitational potential is:

$$U = - \frac{m_p m_c G}{r} \quad (4)$$

The centre of mass is defined by:

$$m_p \underline{r}_1 + m_c \underline{r}_2 = \underline{0} \quad (5)$$

From eqs. (1) and (5):

$$\underline{r}_1 = \frac{m_c}{m_c + m_p} \underline{r} \quad (6)$$

$$\underline{r}_2 = - \frac{m_p}{m_c + m_p} \underline{r} \quad (7)$$

so

$$L = \frac{1}{2} \mu \dot{\underline{r}} \cdot \dot{\underline{r}} + \frac{m_p m_c G}{r} \quad (8)$$

where μ is the reduced mass:

2)

$$\mu = \frac{m_p m_c}{m_p + m_c} - (9)$$

Data Search for the Hulse Taylor Pulsar

1) Google the site large.stanford.edu/course/2007/ph210/ to find "Hulse Taylor pulsar Stanford".

This gives $m_p = 1.42 m(\text{sun}) - (10)$

$$m_c = 1.41 m(\text{sun}) - (11)$$

The semi major axis a is:

$$a = 2.3424 \pm 0.007 \text{ light seconds} - (12)$$

The eccentricity is:

$$e = 0.617155 \pm 0.000007 - (13)$$

Convert to S.I units using:

$$m(\text{sun}) = 1.989 \times 10^{30} \text{ kg} - (14)$$

$$m_p = 2.824 \times 10^{30} \text{ kg} - (15)$$

$$m_c = 2.804 \times 10^{30} \text{ kg} - (16)$$

$$\mu = 1.407 \times 10^{30} \text{ kg} - (17)$$

and

$$- (18)$$

We have:

$$1 \text{ light second} = 2.99792458 \times 10^8 \text{ meter}$$

$$\text{So: } a = 7.02234 \times 10^8 \text{ metres} - (19)$$

The Newton constant is:

$$G = 6.67408(31) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} - (20)$$

and

$$c = 2.99792458 \times 10^8 \text{ m s}^{-1} - (21)$$

3) The half right distance is:

$$d = a(1 - e^2) = 4.34777 \times 10^8 \text{ m} \quad - (22)$$

In the non-relativistic approximation the periastron is defined by

$$r = \frac{d}{1 + e} = 2.6885 \times 10^8 \text{ m} \quad - (23)$$

The periastron is defined by:

$$r = \frac{d}{1 + e \cos \phi} \quad - (24)$$

with $\phi = 0$. $- (25)$

The non-relativistic Lagrangian (\mathcal{L}) describes the motion of a particle μ in a central field described by the potential $U(r)$. The Euler-Lagrange equations are applied to eq. (8) to obtain $\underline{r}(t)$. The individual motions of $\underline{r}_1(t)$ and $\underline{r}_2(t)$ are found using eqs. (6) and (7).

On the non-relativistic level the Euler-Lagrange equations give:

$$\underline{F} = m_p \underline{\ddot{r}} = -m_p m_e b r \frac{\underline{r}}{r^3} \quad - (26)$$

because the potential of interaction of the particle of mass μ with the pulsar of mass m_p is

$$U = -\frac{m_p m_e b}{r} \quad - (27)$$

1) So the non-relativistic equation of motion is:

$$\underline{\ddot{r}} = -m_c G \frac{\underline{r}}{r^3} \quad - (28)$$

The relativistic equation of motion is:

$$\underline{\ddot{r}} = \frac{\gamma m_c G}{r^3} \left(\frac{\underline{v}(\underline{v} \cdot \underline{r})}{c^2} - \underline{r} \right) \quad - (29)$$

where the Lorentz factor is:

$$\gamma = \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \quad - (30)$$

where

$$\underline{v} = \dot{\underline{r}} \quad - (31)$$

Eq. (29) produces orbital precession.

The Einstein theory of general relativity produces the orbital precession:

$$\Delta \phi = \frac{6\pi m_c G}{dc^2} \quad - (32)$$

Using:

$$m_c = 2.804 \times 10^{30} \text{ kg} \quad - (33)$$

$$d = 4.3477 \times 10^8 \text{ m} \quad - (34)$$

It is found that

$$\Delta \phi = 9.028 \times 10^{-5} \text{ rad s}^{-1} \quad - (35)$$

in cases S.I. units.

The observed advance of pulsar's orbital precession is

5) 4.2° a year, as given by the site:

www.astro.cornell.edu/academics/courses/astro201/psr1913.htm
Now assume that the "year" is the earth's year of 365.25 days. Use:

$$\text{One earth year} = 3.154 \times 10^7 \text{ seconds} - (36)$$

and one radian = 2π degrees - (37)

Therefore in one year, the advance is:

$$\Delta\phi = 9.028 \times 10^{-5} \times 3.154 \times 10^7 \times 2\pi \text{ degrees}$$

from the Einstein theory, i.e.

$$\Delta\phi = 17,891 \text{ degrees per year} - (38)$$
$$= 17,891 \text{ degrees per year}$$

Therefore the Einstein theory is incorrect by a factor

of 4,259.8.

To compute the precession from eq. (29) use the initial condition at the periastron:

$$r(0) = \text{periastron} = 2.6885 \times 10^8 \text{ m} - (39)$$

In the excellent non-relativistic approximation the initial velocity at the periastron is:

$$v(0) = mcG \left(\frac{2}{r} - \frac{1}{a} \right) - (40)$$

so
$$v(0) = 1.061 \times 10^5 \text{ m s}^{-1} - (41)$$

2) The Cornell astronomy site cited already give:

$$v(0) = 3 \times 10^5 \text{ m s}^{-1} - (42)$$

Therefore there is a discrepancy of a factor of about 300 between the Cornell and Stanford sites.

There can be no confidence in any claim that the Foster team gives a precise description of the Hulse Taylor binary pulsar.

Finally the orbit of the pulsar shrinks by about 3.1 mm per orbit. This result is again given by the Cornell astronomy site.
