

# 375 (4) : Precession and Spiralling Orbit of ECE2.

Consider the ECE2 Lagrangian:

$$L = -mc^2 \left(1 - \frac{v^2}{c^2}\right)^{1/2} + \frac{GMm}{r} \quad (1)$$

Use of the Euler Lagrange equations in Cartesian coordinates

leads to:

$$\ddot{\mathbf{r}} = \frac{GMm}{r^3} \left( \frac{\mathbf{v}(\mathbf{v} \cdot \mathbf{r})}{c^2} - \mathbf{r} \right) \quad (2)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad (3)$$

The notation of previous page has been used. The major discovery has been made that these equations give a precessing elliptical orbit.

In the non-relativistic limit these equations reduce to:

$$L = \frac{1}{2}mv^2 + \frac{GMm}{r} \quad (4)$$

and

$$\ddot{\mathbf{r}} = -GM\frac{\mathbf{r}}{r^3} \quad (5)$$

Note that the precession from eq. (2) depends only on M. It can be measured graphically and compared with experimental data. In galaxy OJ287 we have of one hundred million sun masses orbits another mass of 18 billion sun masses every twelve years. The orbit precesses by  $39^\circ$  every  $2\pi$  radians. This is an order of magnitude greater than that observed in the

2) solar system.

This experimentally observed precession can be explained by eq. (2) using numerical methods.

The orbit is observed to decrease, and as it goes from 4FT106 to 4FT108 this can be modelled using a potential:

$$U(r) = -mMG \left( \frac{1}{r} + \frac{a}{r^2} \right) \quad (6)$$

where  $a$  is found by comparison with astronomical data. So the initial Lagrangian is:

$$L = -mc^2 \left( 1 - \frac{v^2}{c^2} \right)^{1/2} + mMG \left( \frac{1}{r} + \frac{a}{r^2} \right) \quad (7)$$

The precessing and decreasing orbit can be found from eq. (7) using Cartesian coordinates and computer algebra.

---