Time-dependent x factor

```
(%i1) kill(all);
```
(%o0) *done*

```
(%i1) depends([phi,r,phi_d,r_d],t);
```
(%o1) $[\varphi(t), r(t), \varphi_d(t), r_d(t)]$

```
(%i2) depends(x,[phi,r,t]);
```
(%o2) $[x(\varphi, r, t)]$

1 Eq.(7)

```
(%i3) E7_r: v_r*'diff(v_r,r) + v_phi/r*'diff(v_r,phi);
```
(%o3) $$v_r\left(\frac{d}{d\,r}\,v_r\right)+\frac{v_{phi}\left(\frac{d}{d\,\varphi}\,v_r\right)}{r}$$

```
(%i4) E7_phi: v_r*'diff(v_phi,r) + v_phi/r*'diff(v_phi,phi);
```
(%o4) $$\left(\frac{d}{d\,r}\,v_{phi}\right)v_r+\frac{v_{phi}\left(\frac{d}{d\,\varphi}\,v_{phi}\right)}{r}$$

s is used for switching fluid spacetime on (=1)

```
(%i5) v_r: x*diff(r,t)*s;
```
(%o5) $$\left(\frac{d}{d\,t}\,r\right)s\,x$$

```
(%i6) v_phi: r*diff(phi,t)*s;
```
(%o6) $$\left(\frac{d}{d\,t}\,\varphi\right)r\,s$$

2 v*Del*v

```
(%i7) vDv_r: ev(E7_r,diff);
```
(%o7) $$\left(\frac{d}{d\,t}\,r\right)^2 s^2\,x\left(\frac{d}{d\,r}\,x\right)+\left(\frac{d}{d\,t}\,\varphi\right)\left(\frac{d}{d\,t}\,r\right)s^2\left(\frac{d}{d\,\varphi}\,x\right)$$

```
(%i8) vDv_phi: ev(E7_phi,diff);
```
(%o8) $$\left(\frac{d}{d\,t}\,\varphi\right)\left(\frac{d}{d\,t}\,r\right)s^2\,x$$

3 Eqs. of motion (16,17)

(%i9) `E16: diff(r,t)*diff(x,t)+x*diff(r,t,2)-r*diff(phi,t)^2 + 'vDv_r = -M`

(%o9) $$\left(\frac{d}{d\,t}\,r\right)\left(\frac{d}{d\,t}\,x+\left(\frac{d}{d\,t}\,r\right)\left(\frac{d}{d\,r}\,x\right)+\left(\frac{d}{d\,t}\,\varphi\right)\left(\frac{d}{d\,\varphi}\,x\right)\right)+\left(\frac{d^2}{d\,t^2}\,r\right)x+vDv_r-\left(\frac{d}{d\,t}\,\varphi\right)^2 r=-\frac{G\,M}{r^2}$$

(%i10) `E17: (1+x)*diff(r,t)*diff(phi,t)+r*diff(phi,t,2)+'vDv_phi=0;`

(%o10) $$\left(\frac{d}{d\,t}\,\varphi\right)\left(\frac{d}{d\,t}\,r\right)(x+1)+vDv\_phi+\left(\frac{d^2}{d\,t^2}\,\varphi\right)r=0$$

4 Eqs. of motion (36,37)

(%i11) `E36: ev(E16);`

(%o11) $$\left(\frac{d}{d\,t}\,r\right)\left(\frac{d}{d\,t}\,x+\left(\frac{d}{d\,t}\,r\right)\left(\frac{d}{d\,r}\,x\right)+\left(\frac{d}{d\,t}\,\varphi\right)\left(\frac{d}{d\,\varphi}\,x\right)\right)+\left(\frac{d}{d\,t}\,r\right)^2 s^2\,x\left(\frac{d}{d\,r}\,x\right)+\left(\frac{d}{d\,t}\,\varphi\right)\left(\frac{d}{d\,t}\,r\right)s^2\left(\frac{d}{d\,\varphi}\,x\right)+\left(\frac{d^2}{d\,t^2}\,r\right)x-\left(\frac{d}{d\,t}\,\varphi\right)^2 r=-\frac{G\,M}{r^2}$$

(%i12) `E37: ev(E17);`

(%o12) $$\left(\frac{d}{d\,t}\,\varphi\right)\left(\frac{d}{d\,t}\,r\right)(x+1)+\left(\frac{d}{d\,t}\,\varphi\right)\left(\frac{d}{d\,t}\,r\right)s^2\,x+\left(\frac{d^2}{d\,t^2}\,\varphi\right)r=0$$

5 Limit x --> 1

(%i13) `ev(E36,[x=1]);`

(%o13) $$\frac{d^2}{d\,t^2}\,r-\left(\frac{d}{d\,t}\,\varphi\right)^2 r=-\frac{G\,M}{r^2}$$

(%i14) `ev(E37,[x=1]);`

(%o14) $$\left(\frac{d}{d\,t}\,\varphi\right)\left(\frac{d}{d\,t}\,r\right)s^2+2\left(\frac{d}{d\,t}\,\varphi\right)\left(\frac{d}{d\,t}\,r\right)+\left(\frac{d^2}{d\,t^2}\,\varphi\right)r=0$$

6 Limit x --> 1, s=0

s=0 --> switching fluid spacetime off

(%i15) `ev(E36,[x=1,s=0]);`

(%o15) $$\frac{d^2}{d\,t^2}\,r-\left(\frac{d}{d\,t}\,\varphi\right)^2 r=-\frac{G\,M}{r^2}$$

(%i16) `ev(E37,[x=1,s=0]);`

(%o16) $$2\left(\frac{d}{d\,t}\,\varphi\right)\left(\frac{d}{d\,t}\,r\right)+\left(\frac{d^2}{d\,t^2}\,\varphi\right)r=0$$