A Note on Single Polarization Models

ECE field theory is characterized by non-linear terms $\omega^a_{\mu b} q^b_v$. To get single polarization models, we can suppress the polarization index “a” by multiplying this by $1_a$ and summing over “a” so that

$$\omega^a_{\mu b} q^b_v 1_a = \omega_{\mu b} q^b_v$$

We want to write

$$\omega_{\mu b} q^b_v = \omega_{\mu} q_v$$

or

$$(\omega_{\mu 0} q^0_v + \omega_{\mu 1} q^1_v + \cdots) = (\omega_{\mu})(q^0_v + q^1_v + \cdots)$$

For this to be valid, one requires

$$\omega_{\mu 0} = \omega_{\mu 1} = \cdots$$

where

$$\omega_{\mu b} = \omega^0_{\mu b} + \omega^1_{\mu b} + \cdots = \omega^a_{\mu b} 1_a$$

ECE field theory is characterized by antisymmetry in the spin connection in the $\mu, \nu$ indices,

$$\omega^a_{\mu \mu} = 0$$

or equivalently

$$\omega^a_{\mu b} q^b_\mu = 0$$

As a single polarization model as above, this equation should be of the form

$$(\omega_{\mu 0} q^0_\mu + \omega_{\mu 1} q^1_\mu + \cdots) = (\omega_{\mu})(q^0_\mu + q^1_\mu + \cdots) = 0$$

Three conditions for this to be valid are

$$\omega_{\mu 0} = \omega_{\mu 1} = \cdots = 0$$

or

$$(q^0_\mu + q^1_\mu + \cdots) = 0$$
We won’t consider the pseudo-trivial equation (10) at this time.

It follows from equation (9) that

\[ \omega_{\mu b} q^b_v = 0 \]  \hspace{1cm} (11)

This then effectively removes the non-linear term from the field equations.

However, a third non-unique, non-zero solution to equation (7) can occur if

\[ \det(\{\omega_{\mu b}\}) = 0 \]  \hspace{1cm} (12)

\(\omega_{\mu b}\) is simultaneously defined over two frames of reference making the definition of the determinate questionable.

Given the determinant relation for matrices \(A\) and \(B\),

\[ \det(A B) = \det(A) \det(B) \]  \hspace{1cm} (13)

then

\[ \det(\{\omega_{\mu\nu}\} \{q^\nu_b\}) = \det(\{\omega_{\mu b}\}) = \det(\{\omega_{\mu\nu}\}) \det(\{q^\nu_b\}) \]  \hspace{1cm} (14)

But by the tetrad postulate,

\[ \omega_{\mu\nu} = \Gamma_{\mu\nu} - \partial_\mu q_\nu \]  \hspace{1cm} (15)

thus

\[ \det(\{\omega_{\mu b}\}) = \det(\{\Gamma_{\mu\nu} - \partial_\mu q_\nu\}) \det(\{q^\nu_b\}) \]  \hspace{1cm} (16)

Equation (12) is satisfied if

\[ \det(\{\Gamma_{\mu\nu} - \partial_\mu q_\nu\}) = 0 \]  \hspace{1cm} (17)

Since

\[ \det(\{q^\nu_b\}) \neq 0 \]  \hspace{1cm} (18)

Equation (17) represents a new constraint equation, for a possibly non-trivial single polarization model.