Relativistic Theory of the Conical Section Orbits and Processing Conical Sections

In this case:

$$\frac{d\theta}{d\tau} = \frac{L_0}{mr^2} \quad -(1)$$

where τ is the proper time and where the Lorentz factor is:

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(2)$$

in which the velocity is defined by:

$$\underline{v} = \frac{L_0}{m}\left(\frac{1}{r}\,\underline{e}_\theta - \frac{d}{d\theta}\left(\frac{1}{r}\right)\underline{e}_r\right). \quad -(3)$$

From eq. (1):

$$r^2\, d\theta = \left(1 - \frac{v^2}{c^2}\right)^{1/2} \frac{L_0}{m}\, dt \quad -(4)$$

$$= \frac{L_0}{m}\, d\tau = \frac{1}{\gamma}\frac{L_0}{m}\, dt.$$

Therefore:

$$dt = \frac{m}{L_0}\left(1 - \frac{v^2}{c^2}\right)^{-1/2} r^2\, d\theta \quad -(5)$$

and

$$\boxed{t = \frac{m}{L_0}\int\left(1 - \frac{v^2}{c^2}\right)^{-1/2} r^2\, d\theta} \quad -(6)$$

For the conical sections:

2)

$$r = \frac{d}{1+\epsilon\cos\theta} \quad -(7)$$

and as in note 238(4):

$$v^2 = \left(\frac{L_0}{md}\right)^2\left(1+\epsilon^2+2\epsilon\cos\theta\right) \quad -(8)$$

So:

$$\boxed{t = \frac{m}{L_0}\int\left(1-\left(\frac{L_0}{mdc}\right)^2\left(1+\epsilon^2+2\epsilon\cos\theta\right)\right)^{-1/2}\frac{d^2}{(1+\epsilon\cos\theta)^2}\,d\theta \quad -(9)}$$

This has to be integrated numerically.

The animation proceeds as in note 238(12).

using:

$$x = \frac{d\cos\theta}{1+\epsilon\cos\theta}, \quad -(10)$$

$$y = \frac{d\sin\theta}{1+\epsilon\cos\theta}. \quad -(11)$$

For each θ calculate t from eq. (9).

For example:

$$t_1 = \frac{m}{L_0}\int_0^{\theta_1}\left(1-\left(\frac{L_0}{mdc}\right)^2\left(1+\epsilon^2+2\epsilon\cos\theta\right)\right)^{-1/2}\frac{d^2}{(1+\epsilon\cos\theta)^2}\,d\theta \quad -(12)$$

Repeat for $t_2, t_3, \ldots, t_n$ and animate (x, y) as a function of t.

3) Precessing Conical Sections

In this case:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(13)$$

so

$$t = \frac{m}{L_0}\int\left(1-\frac{v^2}{c^2}\right)^{-1/2}\frac{\alpha^2}{(1+\epsilon\cos(x\theta))^2}d\theta \quad -(14)$$

From eqs. (3) and (13):

$$\underline{v} = \frac{L_0}{m}\left(\frac{(1+\epsilon\cos(x\theta))}{\alpha}\underline{e}_\theta - \frac{d}{d\theta}\left(\frac{1}{r}\right)\underline{e}_r\right) -(15)$$

where:

$$\frac{d}{d\theta}\left(\frac{1}{r}\right) = -\frac{x\epsilon}{\alpha}\sin(x\theta) \quad -(16)$$

so:

$$v^2 = \left(\frac{L_0}{m\alpha}\right)^2\left((1+\epsilon\cos(x\theta))^2 + x^2\epsilon^2\sin^2(x\theta)\right) \quad -(17)$$

Therefore t is calculated from eqs. (14) and (17) for a given θ. For example:

$$t_1 = \frac{m}{L_0}\int_0^{\theta_1}\left(1-\frac{v^2}{c^2}\right)^{-1/2}\frac{\alpha^2}{(1+\epsilon\cos(x\theta))^2}d\theta \quad -(18),$$

$$v^2 = \left(\frac{L_0}{m\alpha}\right)^2\left((1+\epsilon\cos(x\theta))^2 + x^2\epsilon^2\sin^2(x\theta)\right) \quad -(19)$$